How Mathematics can Benefit Agriculture:

The New Zealand Experience

The full version of this talk can be found at

https://connect.innovateuk.org/web/mathsktn/overview

or http://www.minz.org.nz

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FORMULATION

INTEGRATION

Centre for Maths in Industry

FORMULATING SOLUTIONS

INTERPRETATION

SOLUTION

MASSEY UNIVERSITY
TE KUNENGA KI PŪREHUROA

Outline:

1. Introduction:

The under-pinning role of mathematics

2. Examples:

- WATER – Dealing with Pollution.
- EARTH and AIR - Nitrogen and clover cycle.
- FOETAL BIRTH WEIGHT – Optimal nutrient intake
- CONTROLLING PLANT DISEASES - Biocontrol
- PASTURE NUTRIENTS – Optimal allocation

3. Closing remarks:

and pointers to the future

The under-pinning role of Mathematics

Question: Industrial Mathematics Initiatives

Is it an (inter)national need?

OECD Report: The challenge

The Academic Environment

“The academic discipline of mathematics has undergone intense intellectual growth, but its applications to industrial problems have not undergone a similar expansion.”

“
The degree of penetration of mathematics in industry is in general unbalanced, with a disproportionate participation from large corporations and relatively little impact in small- and medium-sized enterprises.”

Key Reference:
Organisation for Economic Co-operation and Development : Global Science Forum Report on Mathematics in Industry July 2008
http://www.oecd.org/dataoecd/47/1/41019441.pdf

This activity has a **positive spin-off**, for it serves to establish better links between industry and academic mathematics.

Plus **enhance the image** of mathematics in the community.

We can provide improved university education of mathematicians through:

- Expanded employment prospects for mathematics graduates.
- Fresh research problems for mathematicians.
- Innovative material for teaching courses.

The lot of an Applied Mathematician

Applied Mathematics seems to be about finding answers to problems. These are not written down in some great book and in reality the hardest task for an applied mathematician is **finding good questions.**

THE TRIVIAL

There seem to be three types of problems in the real world:

THE IMPOSSIBLE

THE JUST SOLVABLE

The boundaries between them are very blurred. They vary from person to person, and some of my strongest memories are of problems that suddenly jump one from category to another, and this is usually with the help of colleagues!

The 10 Commandments

1. **Simple** models do better!
2. **Think** before you compute.
3. A graph is worth **1000** equations.
4. The best computer you've got is **between your ears!**
5. Charge a low fee at first, then **double it** next time.
6. Being wrong is a **step towards** getting it right.
7. Build a (hypothetical) model **before** collecting data.
8. Do experiments where there is *"gross parametric sensitivity"*
9. Learn the biology, etc.
10. **Spend time** on *"decision support"*

2. 1 Water Pollution

Outline

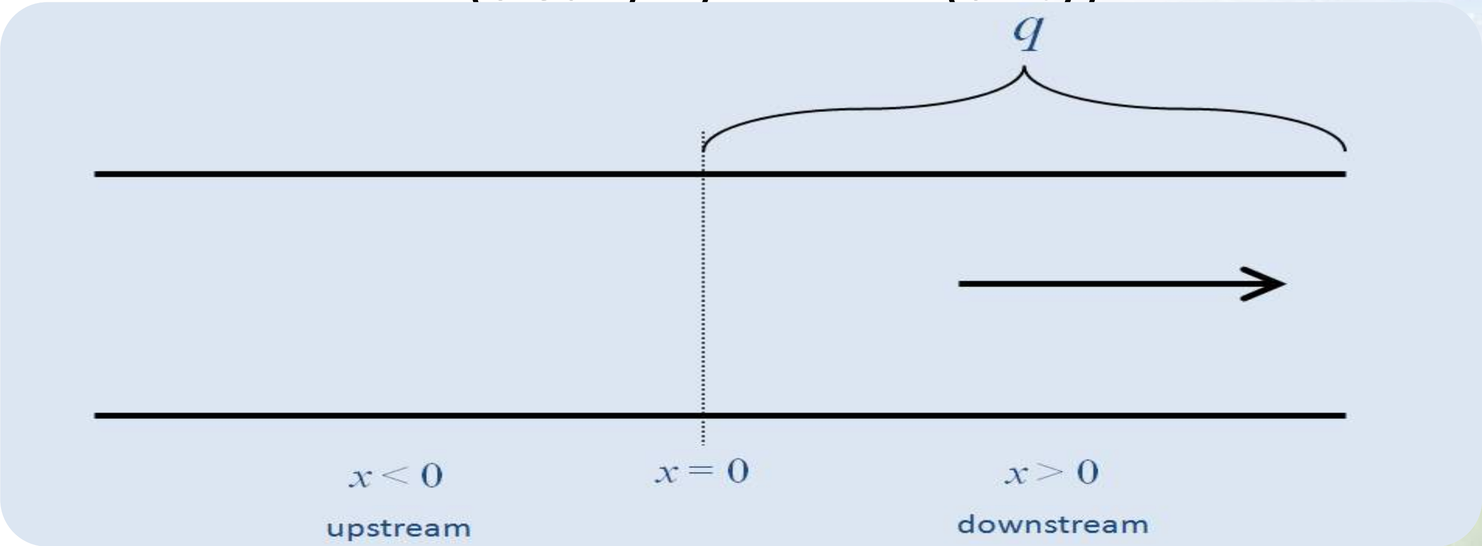
- Motivation –River pollution unsustainable
- The Model description
- Special Cases of the model
- Numerical procedure
- Discussions and conclusions

Motivation

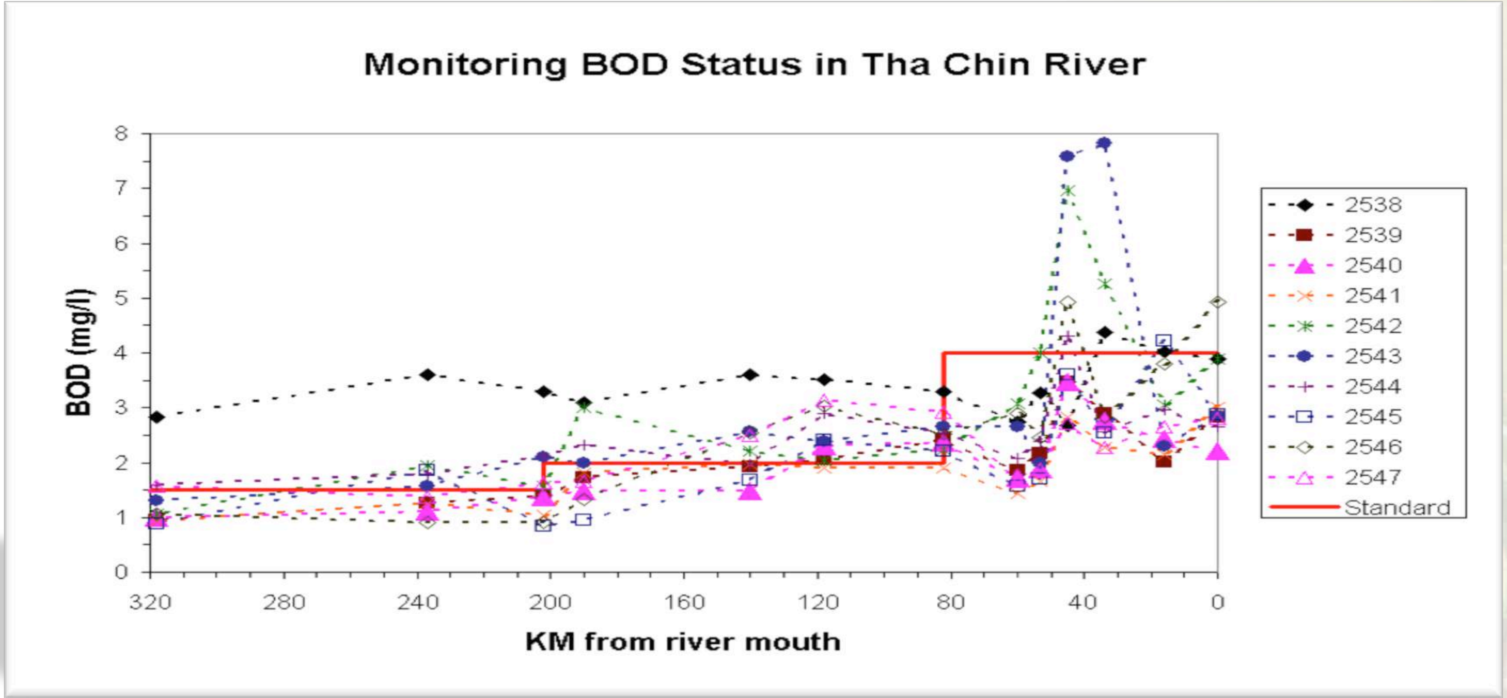
- Economic activities were developed rapidly within the River Basin increasing amounts of contaminants discharge
- Water pollution, through point and non-point sources, have become a major environmental concern in the basin
- Hence, studies of mathematical models of water pollution for this basin are desirable, in order to make effective management of water quality
- The Model

Mathematical model of water quality

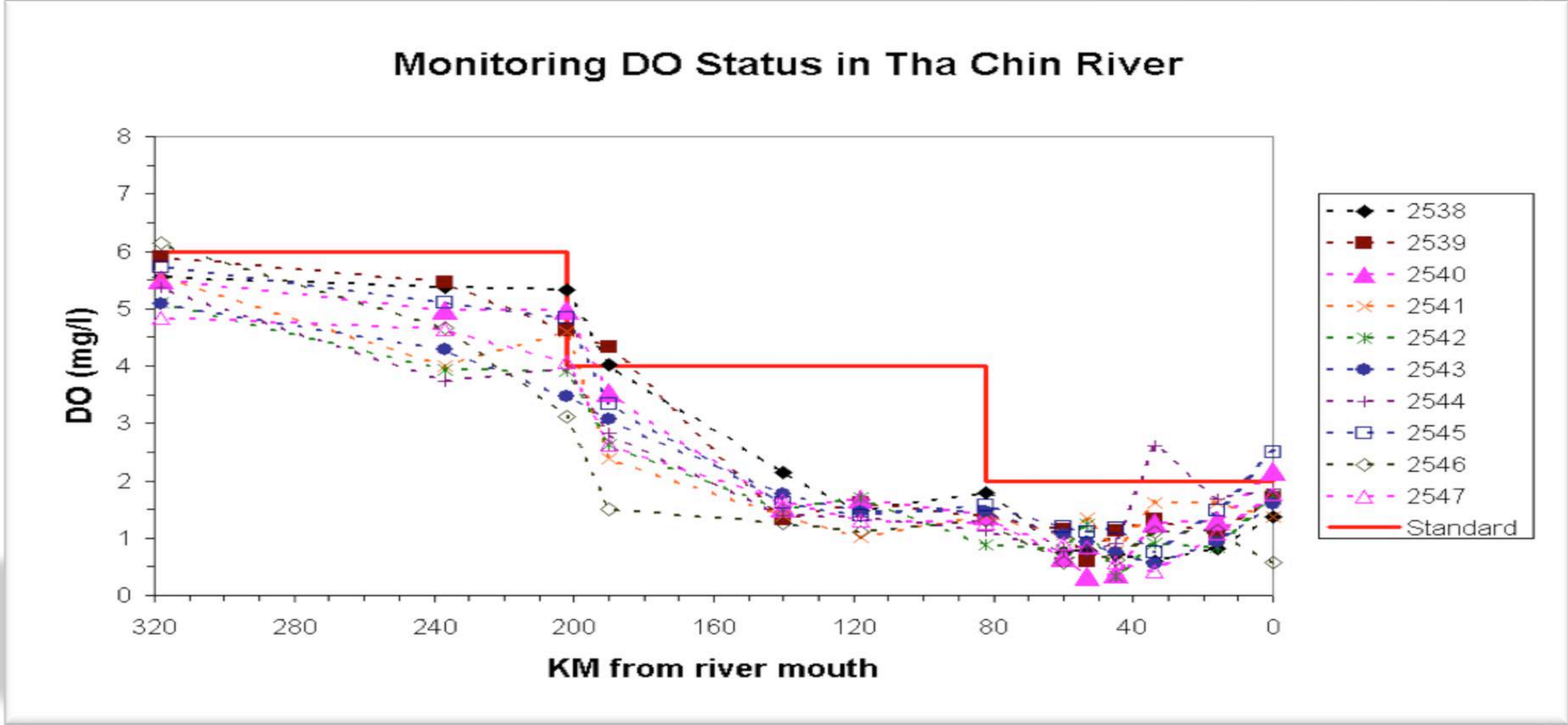
(clean) / (dirty)



Schematic picture of the river & q is the pollutant distributed source



From: Tha Chin River development project: The king of Thailand's initiative: RID



From: Tha Chin River development project: The king of Thailand's initiative, RID

The Mathematics:

Use coupled advection-diffusion equations for

$P(x,t)$: concentration density of pollutant

$X(x,t)$: concentration of dissolved oxygen

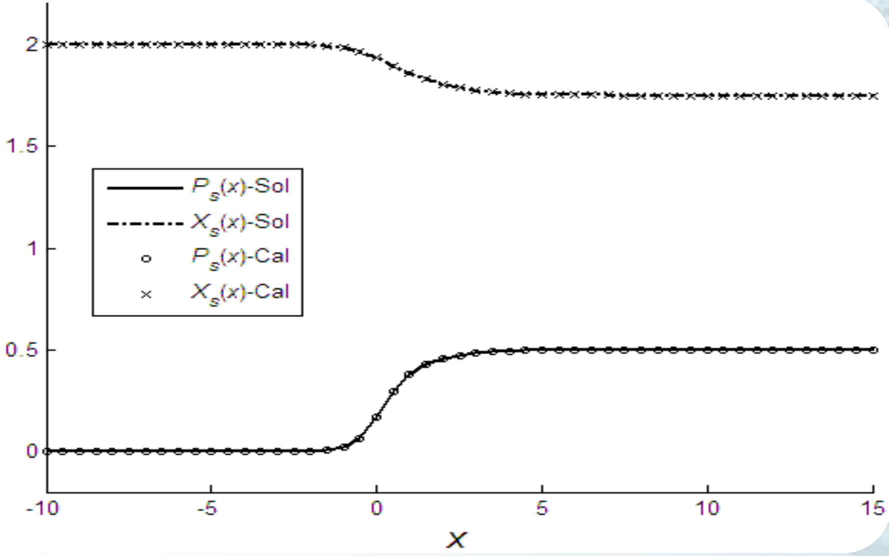
The full model is developed in:

Pimpunchat B, Wake GC, Sweatman WL, Triampo W & Parshotam A) "A mathematical model for pollution in a river and its remediation by aeration"; *Applied Mathematics Letters* **22**: March 2009, pp 304-308.

Numerical procedure reveals:

How transient solutions approach asymptotically to solution downstream

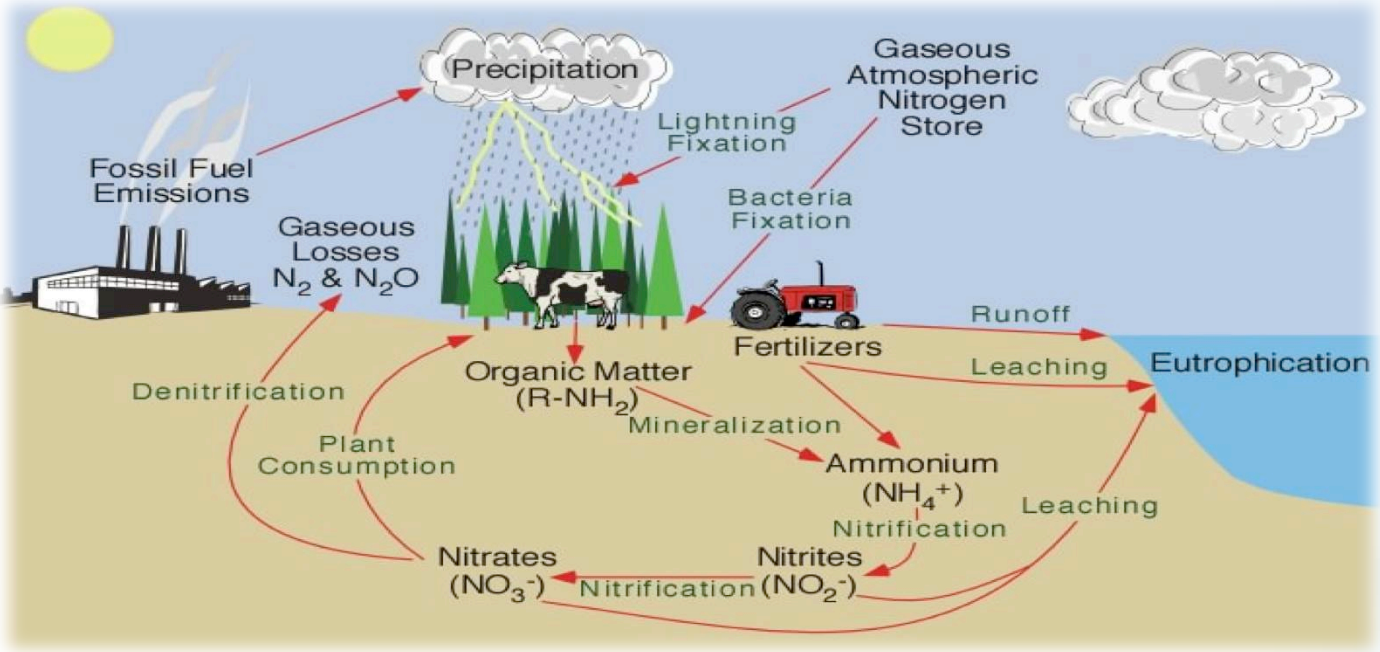
Numerical calculations agree with analytical solution under no pollution and saturated dissolved oxygen far upstream tending to a steady state far downstream for a long river DO depletion in the river.



Concluding remarks

- Such a model and its solutions provides decision support on restrictions to imposed on farming and urban practices.
- The oxygen level fortunately remains above the critical value of 30% of the saturated oxygen concentration and reaches zero far beyond the length of 325 km of Tha Chin River.
- The model appeared capable of illustrating the effect of aeration process to increase DO to the water.
- This constraint is not reached due to the finite length over which pollution is actually discharged and the oxygen concentration which remains above the critical threshold value provided q is low enough.

2.2 MODEL FOR THE NITROGEN CYCLE IN PASTURE



Introduction

Intensively grazed temperate pastures are commonly grass dominant and rate of herbage production is chronically limited by availability of soil mineral nitrogen (N).

For this reason, soil mineral N supply from mineralisation of soil organic matter is often supplemented by regular applications of fertiliser N.

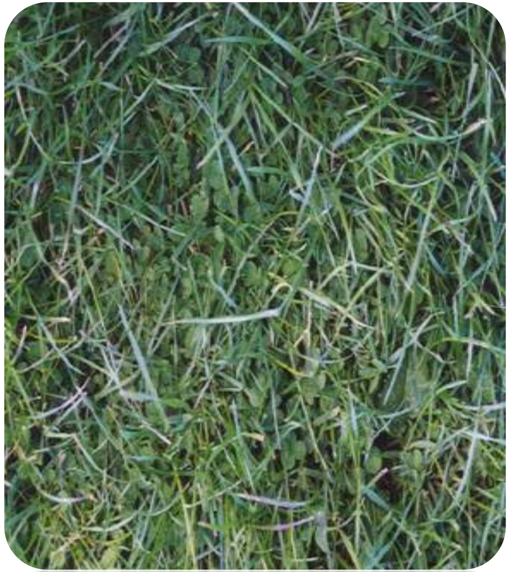
Clovers in these pastures are valued for this N-fixing ability and also for their superior feed quality for grazing animals

However in some countries, including New Zealand, the major inputs to the soil-pasture-animal N cycle are from fixation of atmospheric N_2 by the clover-*Rhizobium* symbiosis.

Rye grass and Clover pasture mix

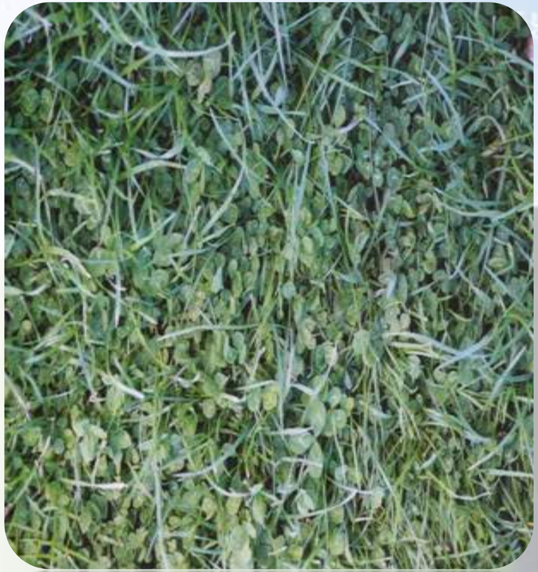
Clover Percentages

10%

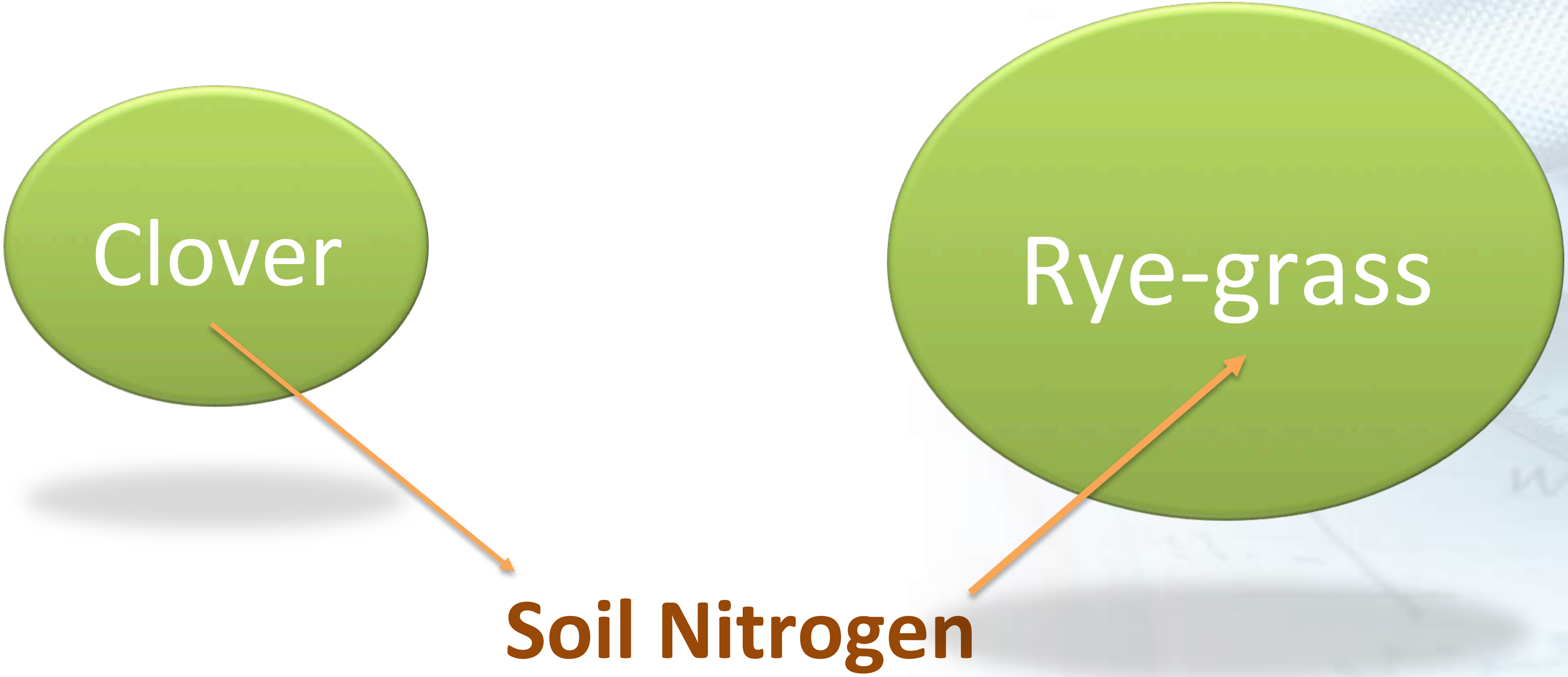


20%

40%



Enhancement of grass growth



Television interview

Dr Andy West – May 2008 (Ex-CEO of AgResearch)

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agresearch

“New Zealand's largest Crown Research Institute

A combination of renowned research centres such as Ruakura, Grasslands, Lincoln and Invermay.

But most importantly, individual scientists and teams whose work is at the heart of pastoral industries, food processing and innovative products that benefit all New Zealand

Mathematics has had a strong focus in this work, with a major underpinning role.”

Distributed-delay-differential equations

(These arise when the past affects the present)

Example 1: $y'(t) = y(t-T)$, $y([-T,0])$ **given: Point delay**

What is the solution?? Homework!

(Often the past effects might be distributed)

Example 2: $y'(t) = r\, y(t)\,\left(1 - \int_0^t (y(s)/K)\, ds\right)$ $y(0) = y_o$

Logistic DDE. What is the solution??

Model development

Clover content in pastures is commonly considered less than optimum, and is subject to large fluctuations in time.

The proportion of clover in pasture and rate of N fixation has large environmental impact for farmers and the wider community.

Soil N availability has a powerful effect on clover/grass proportions, clovers being more competitive where N availability is low while grasses dominate where N availability is high in part because uptake of N from the soil is more efficient than a combination of uptake and fixation.

However there are also intrinsic reasons for this variability, relating to the interactions between coexisting clovers and grass.

The observed variability over time in clover content is often attributed to environmental factors e.g., plant mortality due to adverse climate or pests and diseases.

Formulation

The state variables are:

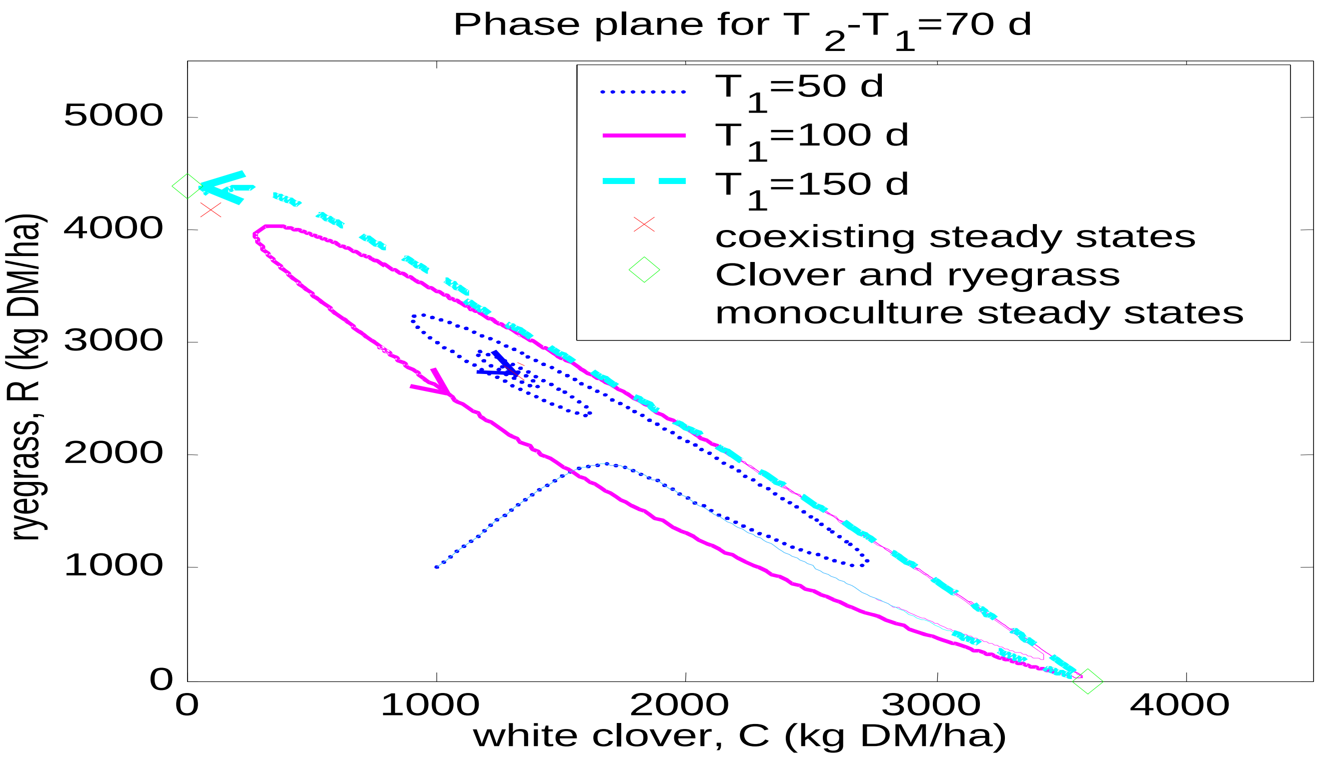
- **Standing masses at time *t* of clover *C(t)*, and ryegrass *R(t)***
- **These will be measured in units of kg DM (dry matter) per hectare**

We do not attempt to describe the nitrogen cycling explicitly since this would require introducing a variable for soil nitrogen, which is extremely difficult to measure.

We introduce a gross simplification of the complex processes which occur over various timescales when clover “fixes” nitrogen in an attempt to capture the essence of the fixation mechanism.

These processes include growth of clover tissue from the fixed nitrogen, senescence of this tissue and subsequent mineralisation which makes the nitrogen available in a form that can be used for ryegrass growth.

Instead of describing these processes explicitly, we relate the enhancement effect on ryegrass growth to the total amount of clover mass turnover occurring in a past period of time.


Phase plane for T 2-T 1=70 d
T 1=50 d
T 1=100 d
T 1=150 d
coexisting steady states
Clover and ryegrass
monoculture steady states
5000
4000
3000
2000
1000
0
ryegrass, R (kg DM/ha)
0
1000
2000
3000
4000
white clover, C (kg DM/ha)

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Conclusions

Feature of the model is an N fixation mechanism that attempts to describe the beneficial effect of clover on ryegrass without explicitly introducing an N variable.

Local analysis of the model together with some simulations using the delay times T_1 and T_2 as the control parameters

This leads to a wide variety of outcomes.

These range from monocultures of clover and ryegrass, to coexistence equilibria, to periodic solutions where the period can be as large as 10 years.

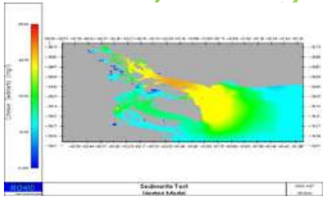
In these periodic solutions it is possible to have long time intervals where both clover and ryegrass standing mass are alternately low and high but the oscillations are sustained indefinitely.

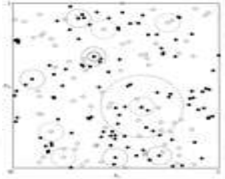
This is in contrast to the simulations reported by Thornley *et al*. (1995) where large-scale fluctuations existed but eventually settled into a coexistence equilibrium.

Similarly, the spatial model simulations of Schwinning & Parsons (1996) suggested the amplitude of the oscillations declined with time.

The delays introduced the oscillatory behaviour, which is observed *(as with the simple delay-logistic)*. **A key decision tool is here.**

Reference: K Louie, GC Wake, MG Lambert, A McKay & D Barker "A delay model for the growth of rye-grass-clover mixtures: Formulation and preliminary simulations", *Ecological Modelling*, **155,** 2002, pp31-42.





2.3 OPTIMAL NUTRITIONAL STRATEGIES FOR MAMMALIAN DEVELOPMENT: A SYSTEMS APPROACH

Graeme Wake and Chanakarn Kiataramkul

(Thailand)

Introduction

Pregnancy is the main event in the life of a female mammal to reproduce its progeny and maintain the integrity of the species.

The process of the feotal development is started as a result of fertilization. After that, a zygote and then an embryo will form. The embryo is implanted in the uterine cavity of female mammal and develops into a fetus.

In this study, we are interested in a specific mammal, namely the sheep. The gestation length in sheep varies from 142 to 152 days. The average is 147 days. There can be multiple gestations, as in the case of twins or triplets.

The fetus receives nutrition from the mother critically in the late gestation, during which fetal growth takes place. Nutrition intake of the mother affects the size and strength of lambs and the milk producing ability of the ewe.

Biological studies have shown that both maternal undernutrition and overnutrition during pregnancy have an impact on feotal growth and development, resulting in an increase in the fetal and neonatal mortality and morbidity.

We consider the fetal growth in a singleton pregnancy of sheep in the second half of pregnancy, from day 75 to day 147.

We use the birth weight as an indicator of the health of both the fetus and the mother. Our goal is to find the daily nutrient intake that will achieve a desirable birth weight while minimizing the total nutrient intake in the second half of the pregnancy.

Optimal Control Problem

The fundamental problem in optimal control theory is to determine a feasible control that maximizes (or minimizes), where *u(t)* is nutrient intake rate of the mother, and *x(t)* is the current fetal mass….

$$J\{u\} = \int_{t_0}^{t_b} g\left[t, x(t), u(t)\right] dt$$

subject to $$\frac{d}{dt} x(t) = f\left(t, x(t), u(t)\right)$$

with the boundary conditions: $x(t_0) = x_0$ and $x(t_b) = x_b$

We can generate the necessary conditions from the Hamiltonian H, which is defined as follows:

$$H(t,x,u,\lambda) = g(t,x,u) + \lambda f(t,x,u)$$

The necessary conditions can be written in terms of the Hamiltonian, using Pontryagin control theory:

$$\frac{\partial H}{\partial u} = 0 \text{ at } u^* \Rightarrow g_u + \lambda f_u = 0 \qquad \text{(optimality condition),}$$

$$\dot{\lambda} = -\frac{\partial H}{\partial x} \Rightarrow \dot{\lambda} = -(g_x + \lambda f_x) \qquad \text{(adjoint equation),}$$

$$\dot{x} = f(t,x,u) = \frac{\partial H}{\partial \lambda} \qquad \text{(state equation)}$$

Our objective functional minimizes the whole nutrient intake for the second half of the pregnancy with respect to a system of ordinary differential equations modeling fetal growth.

Minimize $J\{u\} = \int_{t_0}^{t_b} u \, dt$ = Total nutrient intake

subject to $\frac{dx}{dt} = f(x, u)$ = Growth function

with the boundary conditions: $x(t_0) = x_0$ and $x(t_b) = x_b$

Fetal Growth Model

Table 1: The values of parameters *r*, *K* and the calculated R^2 for the linear, exponential, logistic and Gompertz functions with $x_0 = 0.2$ kg.

Function	$x(t)$	r	K	R^2
Linear	$x_0 + rt$	0.06	-	0.892
Exponential	$x_0 e^{rt}$	0.05	-	0.590
Logistic	$\dfrac{x_0 K}{x_0 + (K - x_0)e^{-rt}}$	0.07	7	0.983
Gompertz	$x_0 e^{\left\{\ln\left(\frac{K}{x_0}\right)\left(1-e^{-rt}\right)\right\}}$	0.02	15	0.978

To obtain the appropriate logistic function which best fits our experimental data, we use the least squares method with the following logistic equation,

$$x(t) = \frac{x_0 K}{x_0 + (K - x_0)e^{-rt}}$$

We then use the two-dimensional Newton's method to estimate the parameters *r* and *K*. We found that $r \cong 0.07$ days^{-1} and $K \cong 7$kg.

To include the nutritional intake as a control in the logistic function, we multiply r by the Michaelis-Menten relationship, $\frac{u}{u+L}$.

So our optimal control problem becomes:

Minimize $$J\{u\} = \int_{t_0}^{t_b} u \, dt$$

subject to $$\frac{dx}{dt} = \frac{rxu}{u+L}\left(1 - \frac{x}{K}\right)$$

with the boundary conditions: $x(0) = 0.2$ and $x(72) = 5.5$

After modifying the model to make it more realistic, taking into account the fact that the history of nutrient intake plays an important role in fetal growth, the carrying capacity (K) is represented by a prescribed function of the cumulative intake using empirical relationships suggested by data analysis.

$$K(t) = K_0 + \left(\frac{ay}{y + L_0} \right)$$

$$y(t) = \int_0^t e^{-\beta(t-s)} u(s)\, ds$$

where

Thus, our optimal control problem becomes:

Minimize $$J\{u\} = \int_{t_0}^{t_b} u\,dt$$

subject to $$\frac{dx}{dt} = \frac{rxu}{u+L}\left(1 - \frac{x}{K_0 + \frac{ay}{y+L_0}}\right)$$

$$\frac{dy}{dt} = u - \beta y$$

with the boundary conditions: $x(0) = 0.2$, $x(72) = 5.5$ and $y(0) = 0$.

Solution of One-Dimensional Optimal Control Problem

The Pontryagin's Maximum Principle gives a Hamiltonian expression:

$$H\left(t, x, u, \lambda\right) = u + \frac{rxu\lambda}{u+L}\left(1 - \frac{x}{K}\right)$$

Optimality Condition: $\frac{\partial H}{\partial u} = 0 \text{ at } u^*$

$$u^* = -\frac{KL \pm \sqrt{Krx^2\lambda L - rx\lambda K^2 L}}{K}$$

Adjoint Equation:

$$\dot{\lambda} = -\frac{\partial H}{\partial x}$$

$$\dot{\lambda} = \frac{ru\lambda}{u+L}\left(\frac{2x}{K} - 1\right)$$

Dynamical System:

$$\dot{x} = \frac{rxu}{u+L}\left(1 - \frac{x}{K}\right)$$

$$\dot{\lambda} = \frac{ru\lambda}{u+L}\left(\frac{2x}{K} - 1\right)$$

with the boundary conditions: $x(0) = 0.2$ and $x(72) = 5.5$

We obtain the formula for $\dot{u}$:

$$\dot{u} = \frac{g_x f_u^2 - g_{ux} f f_u - g_u f_x f_u + g_u f_{ux} f}{g_{uu} f_u - g_u f_{uu}}$$

Dynamical System:

$$\dot{x} = \frac{rxu}{u + L}\left(1 - \frac{x}{K}\right)$$

$$\dot{u} = 0$$

with the boundary conditions: $x(0) = 0.2$ and $x(72) = 5.5$

Solution of Two-Dimensional Optimal Control Problem

The Pontryagin's Maximum Principle gives a Hamiltonian expression:

$$H = u + \lambda_1 \frac{rux}{u+L}\left(1 - \frac{x}{K_0 + \dfrac{ay}{y+L_0}}\right) + \lambda_2 (u - \beta y)$$

$$u^* = -L \pm \frac{\sqrt{\left(K_0 + \dfrac{ay}{y+L_0}\right)\lambda_1 rLx\left(x - \left(K_0 + \dfrac{ay}{y+L_0}\right) + \lambda_2 x - \lambda_2\left(K_0 + \dfrac{ay}{y+L_0}\right)\right)}}{\left(K_0 + \dfrac{ay}{y+L_0}\right)(1+\lambda_2)}$$

Dynamical System:

$$\dot{x} = \frac{rxu}{u+L}\left(1 - \frac{x}{K_0 + \frac{ay}{y+L_0}}\right)$$

$$\dot{y} = u - \beta y$$

$$\dot{\lambda}_1 = \frac{-\lambda_1 ru}{u+L}\left(1 - \frac{2x}{K_0 + \frac{ay}{y+L_0}}\right)$$

$$\dot{\lambda}_2 = \frac{-\lambda_1 arL_0ux^2}{\left(K_0 + \frac{ay}{y+L_0}\right)^2 (u+L)(y+L_0)^2} + \beta\lambda_2$$

with the boundary conditions: $x(0) = 0.2,\ x(72) = 5.5,\ y(0) = 0$ and $\lambda_2(72) = 0.$

Theorem 3.1: If $\dot{x} = f(t,x,u) = f_1(x) \cdot f_2(u)$ is separable and *g* is not a function of *x*, $g = g(t,u)$ for which $J\{u\} = \int_{t_0}^{t_b} g\left[t, u(t)\right] dt$ then $\dot{u} = G(t,x,u) \equiv 0$, so that *u* is a constant.

Proof: Assume that

$$\dot{x} = f(t,x,u) = f_1(x) \cdot f_2(u).$$

It has been shown that $\dot{u} = \dfrac{g_x f_u^2 - g_{ux} f f_u - g_u f_x f_u + g_u f_{ux} f}{g_{uu} f_u - g_u f_{uu}}$

Since $g = g(t,u)$, then $g_x = g_{ux} = 0$, so

$$\dot{u} = \frac{-g_u \dot{f}_1(x) f_2(u) f_1(x) \dot{f}_2(u) + g_u \dot{f}_1(x) \dot{f}_2(u) f_1(x) f_2(u)}{g_{uu} f_1(x) \dot{f}_2(u) - g_u f_1(x) \ddot{f}_2(u)} = 0$$

Numerical Results for One-Dimensional Optimal Control Problem

We solve the dynamical system, which uses the original and alternative optimal control algorithm , with the following parameters and boundary conditions: $t_0 = 0$ days, $t_b = 72$ days, $x(0) = 0.2$ kg, $x(72) = 5.5$ kg, , $r = 0.07 \text{ days}^{-1}$, $K = 7$ kg. and $L = 0.09 \text{ kg} \cdot \text{days}^{-1}$

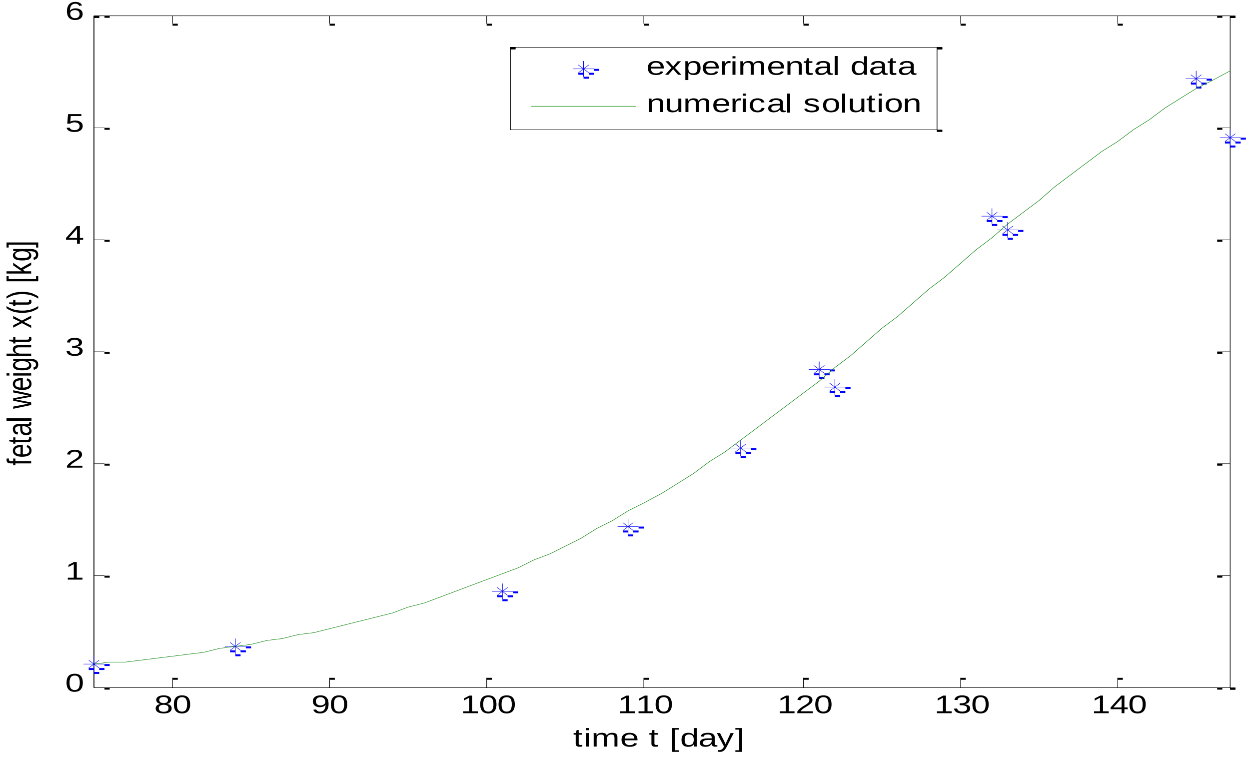


Figure 2: The sheep fetal weight over the 72 days of the second half of the pregnancy, plotted together with the experimental data.

The calculated fetal weight fits the data very well with the coefficient of determination, R^2 of 0.98528.

By solving this system, we obtain the nutrient intake, considered as the control u, to be a constant which equals $2.0254\,\mathrm{kg}\cdot\mathrm{days}^{-1}$ and the total intake for the mother in the second half of the pregnancy is

$$\int_0^{72} u\,dt = 145.8787\ \mathrm{kg}$$

Numerical Results for Two-Dimensional Optimal Control Problem

We solve the dynamical system, which uses the original and alternative optimal control algorithm , with the following parameters and boundary conditions: t_0 = 0.2 days, t_b = 5.5 days, $x(0)$ = 0.2 kg, $x(72)$ = 5.5 kg, $y(0)$ = 0, $\lambda_2(72)$ = 0, r = 0.07 days^{-1}, L = 0.09 kg· days^{-1}, K_0 = 7 kg, a = 0.1 kg, β = 0.12 days^{-1} and L_0 = 10 kg.

Why x(0) is so low?

FETAL DEVELOPMENT

From zygote to full term.

For McGraw-Hill Publishing

© Cynthia Turner

We consider the coefficient of determination, R^2, which represents how well the experimental data of fetal weight fits the computed feotal weight, to determine which parameter values are appropriate for the model.

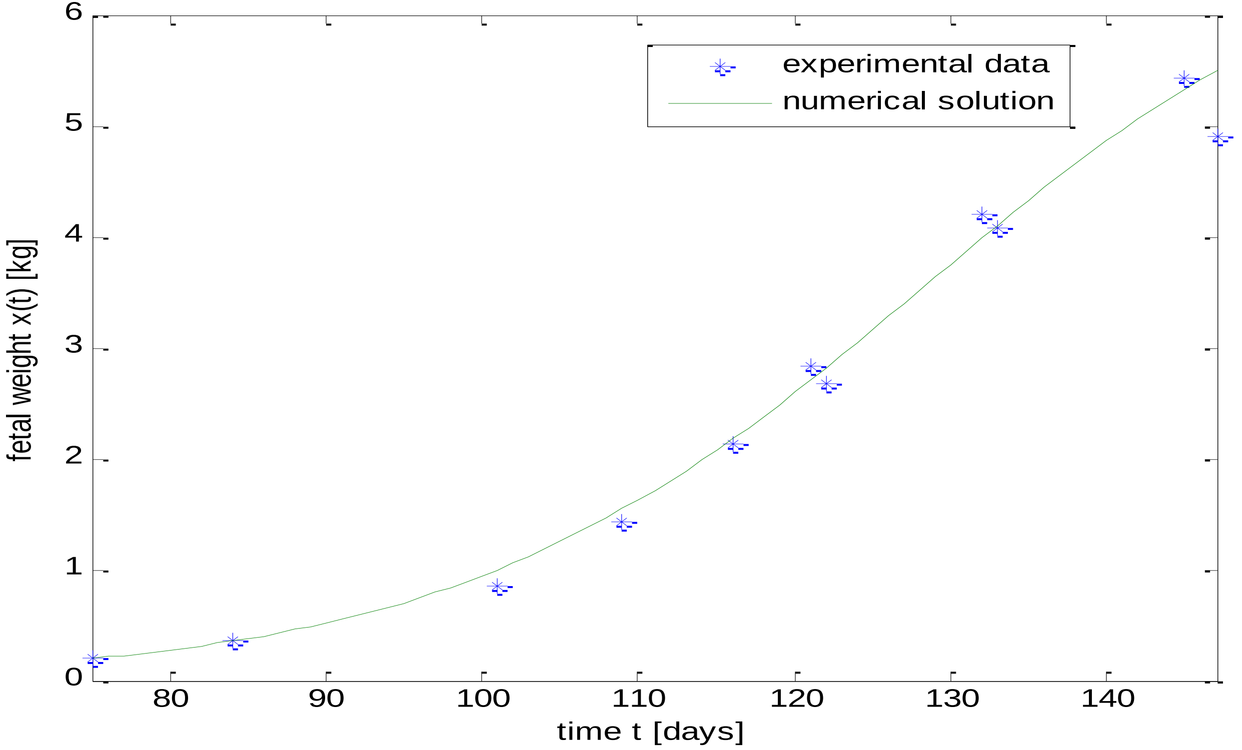


Figure 3: The sheep feotal weight over the 72 days of the second half of the pregnancy, plotted together with the experimental data from solving the dynamical system, $R^2 = 0.98541$.

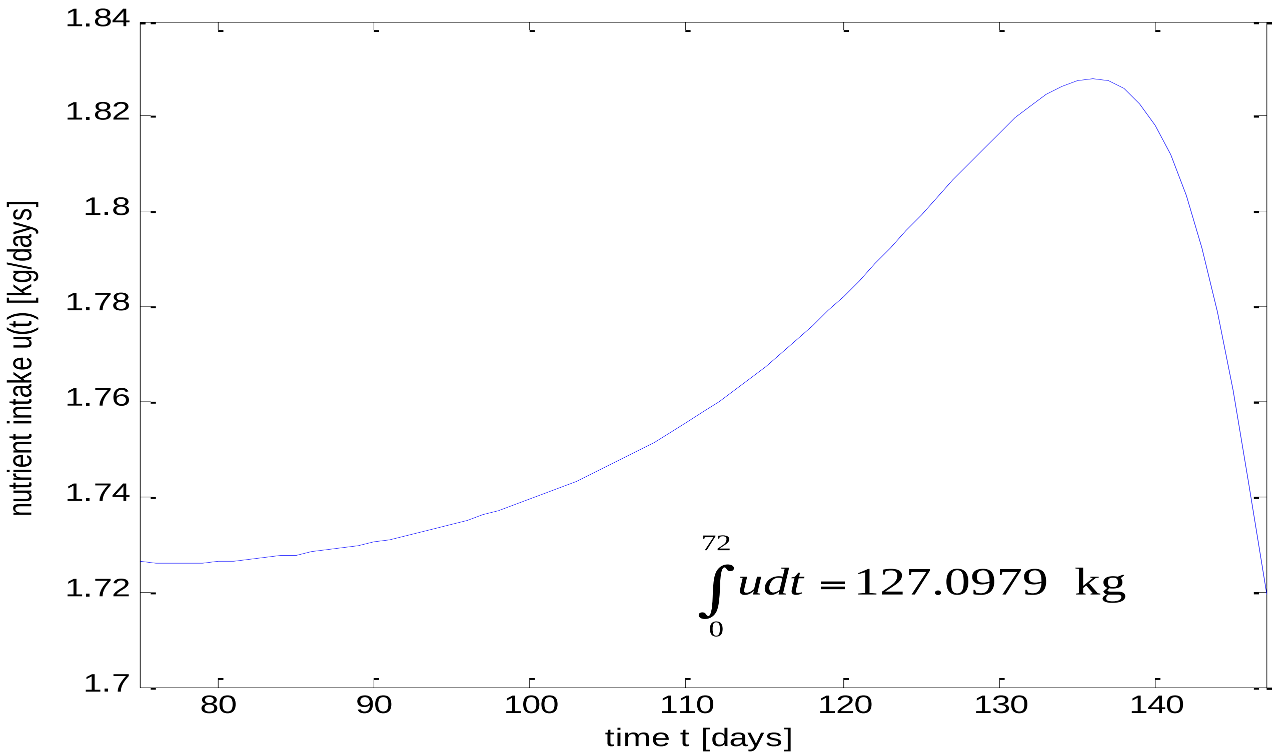


Figure 4: The minimum daily nutrient intake over the 72 days of the second half of the pregnancy to achieve the pre-set birth weight.

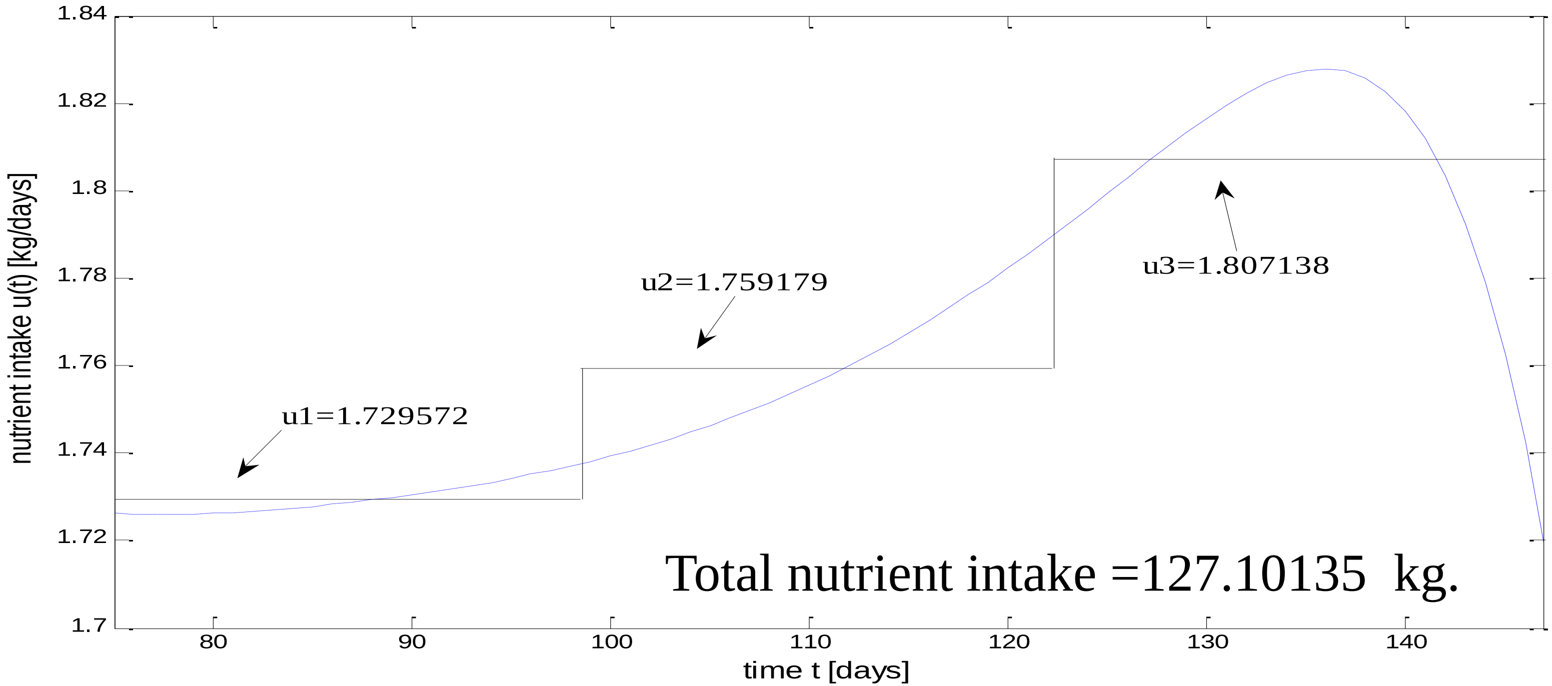


Figure 5: The minimum daily nutrient intake over 72 days of the second half of pregnancy plotted together with the sub daily nutrient intakes in three equal subintervals of time.

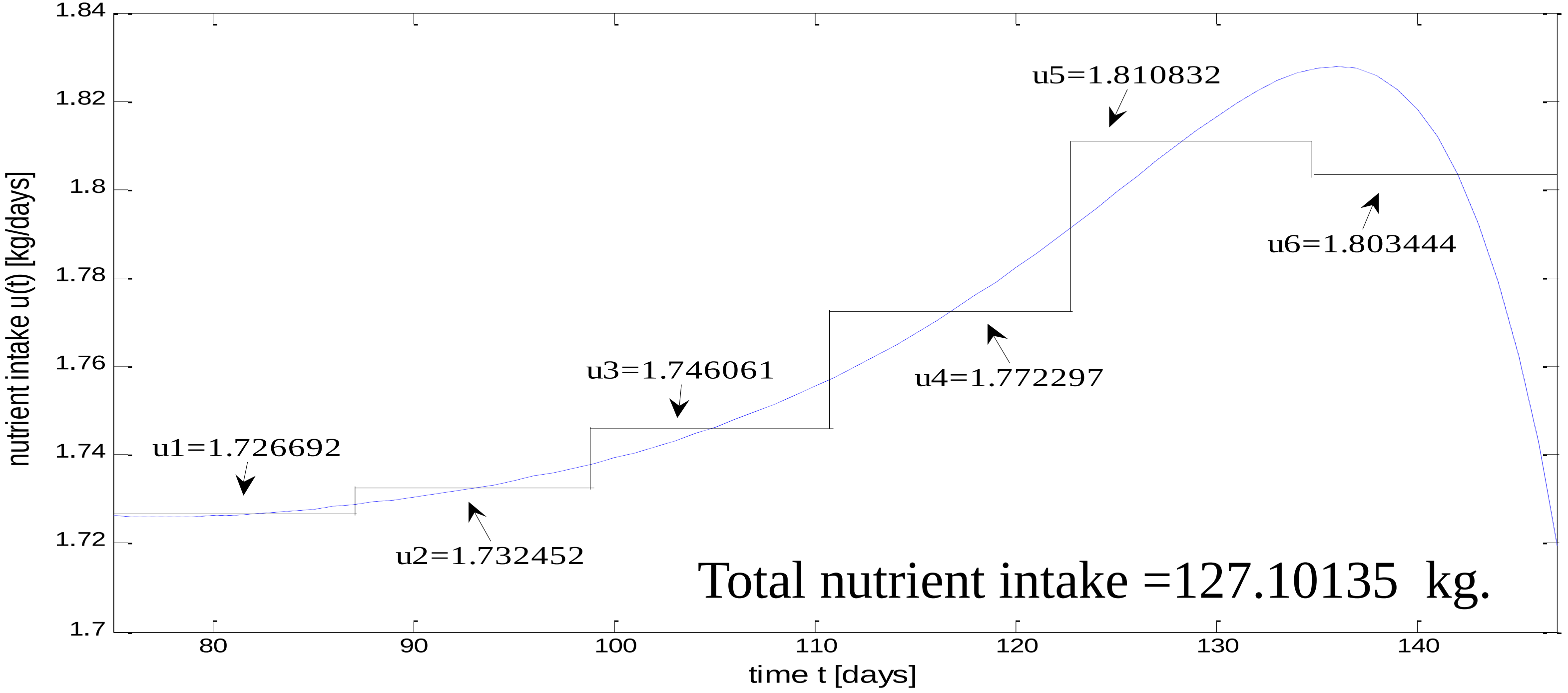


Figure 6: The minimum daily nutrient intake over 72 days of the second half of pregnancy plotted together with the sub daily nutrient intakes in six equal subintervals of time.

Discussion and Conclusion

We have considered the feotal growth in a singleton pregnancy of sheep during the second half of pregnancy after the differentiated fetus has been formed, from day 75 to day 147. We use the birth weight as an indicator of the health of both the feotus and the mother.

Our goal was to find the daily nutrient intake that would achieve a desirable birth weight while minimizing the total nutrient intake in the second half of the pregnancy.

Finding the optimal nutrient intake to reach the desirable birth-weight increases the quality of post-natal life-history and gives rise to a better economic output for farmed animals.

The optimal control strategy has been used in our problem, in which the end point is set.

We derived the alternative algorithm for a direct calculation of the control variable *u*, which now appears explicitly as a component function in the dynamical system, without having to calculate all the solutions of the adjoint equation(s).

The numerical solutions with specific parameter values have been obtained. We also show numerical results on the nutrient intake as a step function by dividing the total time equally into three and six periods. We then obtain the nutrient intake in each of the three or six time periods so that it is easier to advice a farmer on how to feed the animals in a real situation.

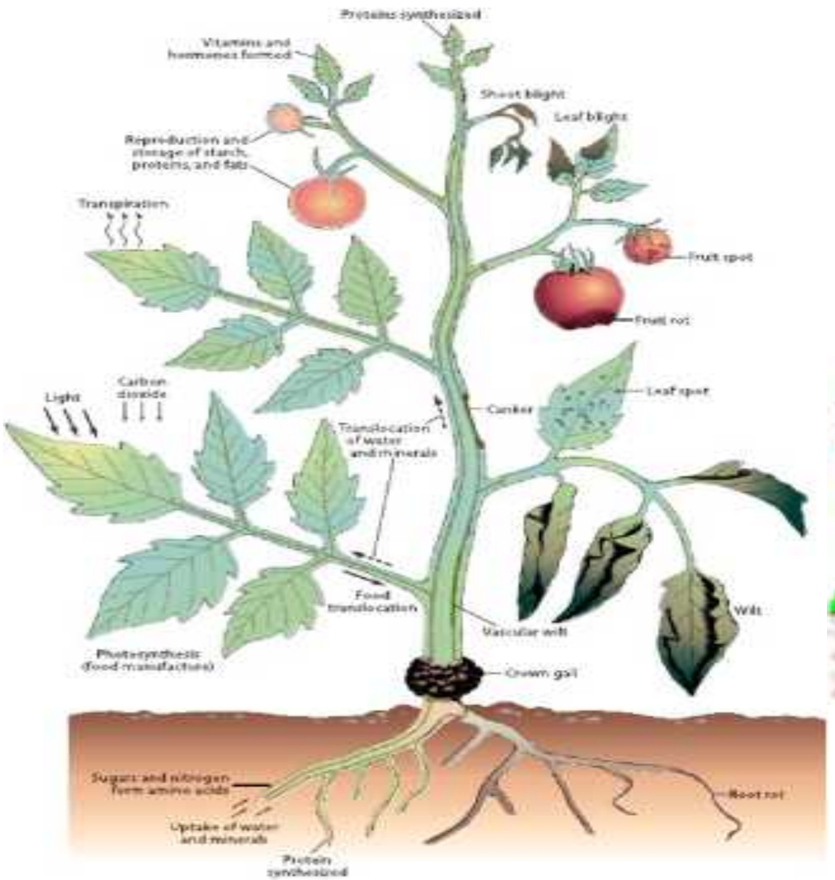
A twin or triple pregnancy, or pregnancy in other mammals, can be considered and the same algorithm may be successfully applied.

Mathematical Model of Induced Resistance to Plant Diseases

Dr Nurul Syaza Abdul Latif

Prof. Graeme Wake (Massey University, New Zealand)
Dr. Philip Elmer (Plant & Food Research, Hamilton, New Zealand)
Dr. Tony Reglinski (Plant & Food Research, Hamilton, New Zealand)

Mathematical Model of Induced Resistance to Plant Diseases
I—Introduction to Plant Disease



Definition:

A plant in general become diseased when it is continuously disturbed by causal agents that results in physio-logical abnormality, that is disruption of the plant's normal structure and growth. Causal agents:

Abiotic

Biotic

Schematic representation of the basic functions in a plant (left) and the interference with these functions (right) caused by some common types of plant diseases (source: Agrios (2005)).

Fruit rot

Shoot blight

Examples of Plant Disease

Fruit spot

Leaf spot

Plant Disease Management

Eradication

Exclusion

Plant Disease Management

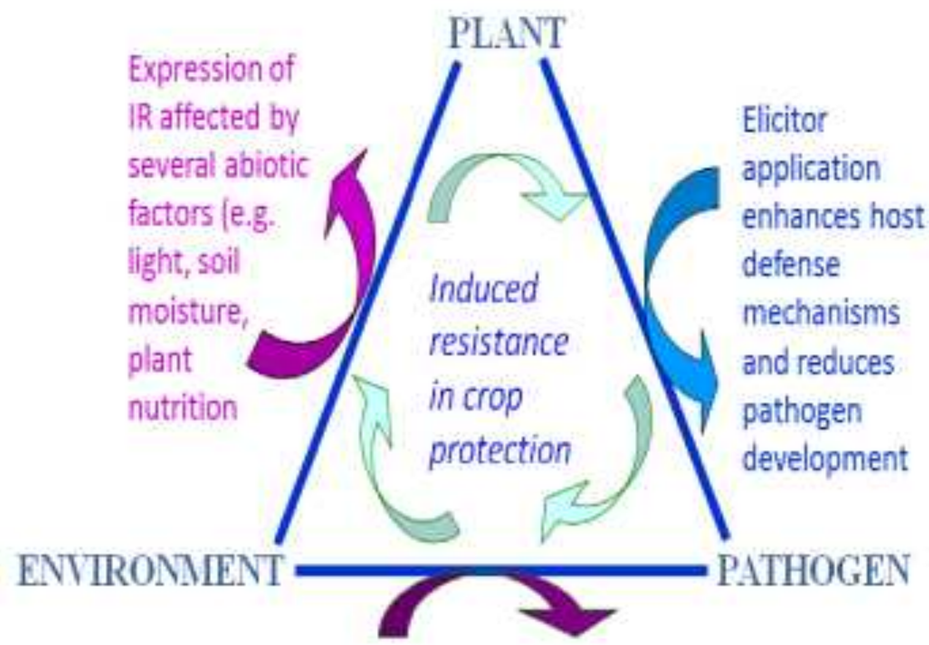
Protection

Resistance

I—Induced Resistance
^—What is IR?

IR pathogen: defence responses that occur after the plant is challenged by the pathogen or by an externally applied compound (elicitor).

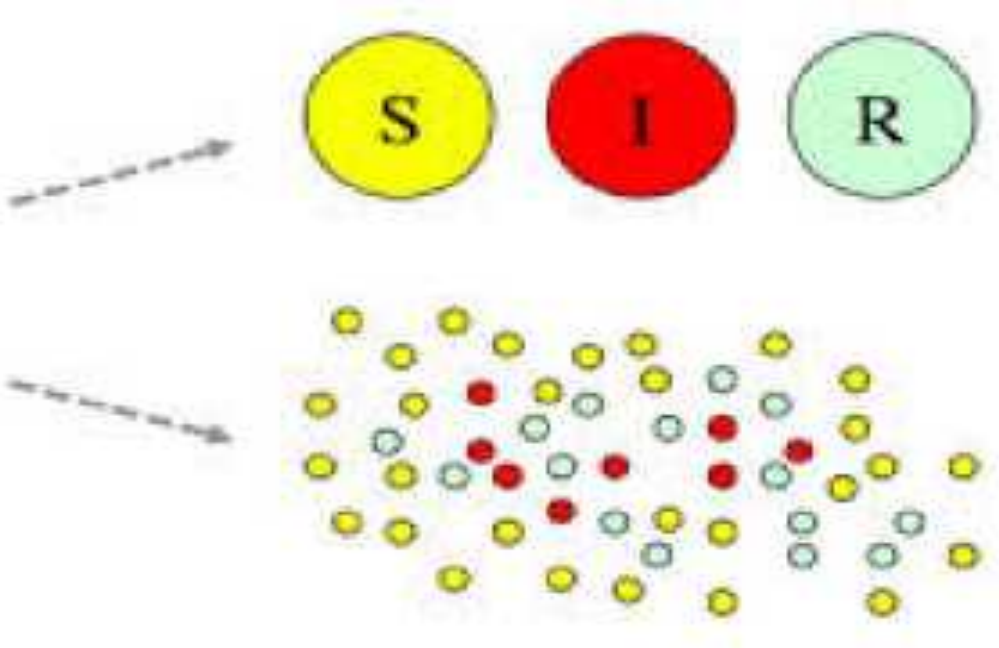
- application of elicitor has shown to activate defence genes and protect the plants against disease.
- IR can provide 20-85% disease control
- IR is affected by many factors: especially the environment.



Favourable conditions may increase disease development
And reduce IR response

Mathematical Model of Induced Resistance to Plant Diseases
^—Development of a Prototype Induced Resistance Model
I—Model’s Assumptions

This IR Model is based on Susceptible-Infected-Remove Model and basic Differential Equations:



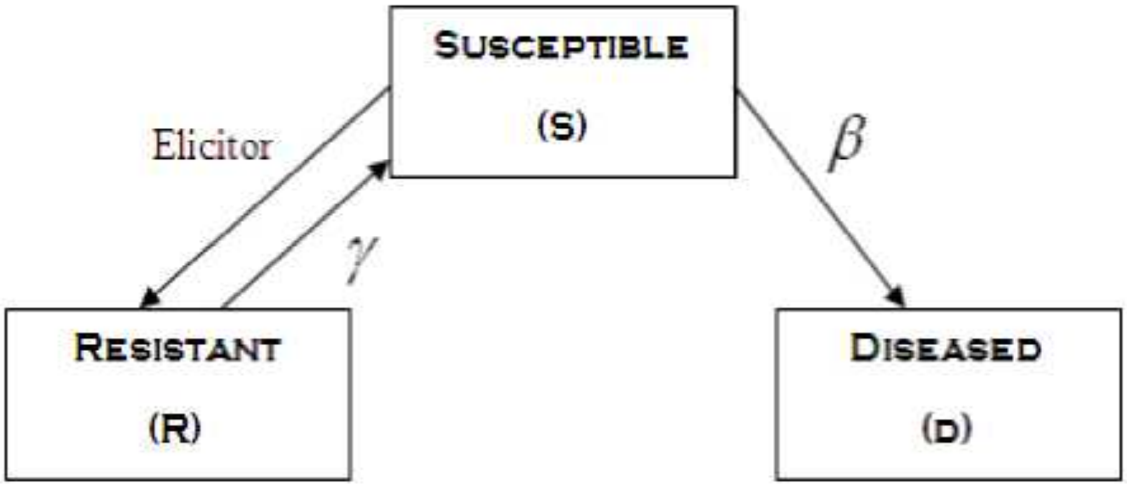


Figure: A simplified phenomenon of induced resistance in the treated plants. For the untreated plants, there is only one way flow which is from S to R and described as a SIR-model.

Mathematical Model of Induced Resistance to Plant Diseases

└─Development of a Prototype Induced Resistance Model

  └─Induced Resistance Experiment Timeline

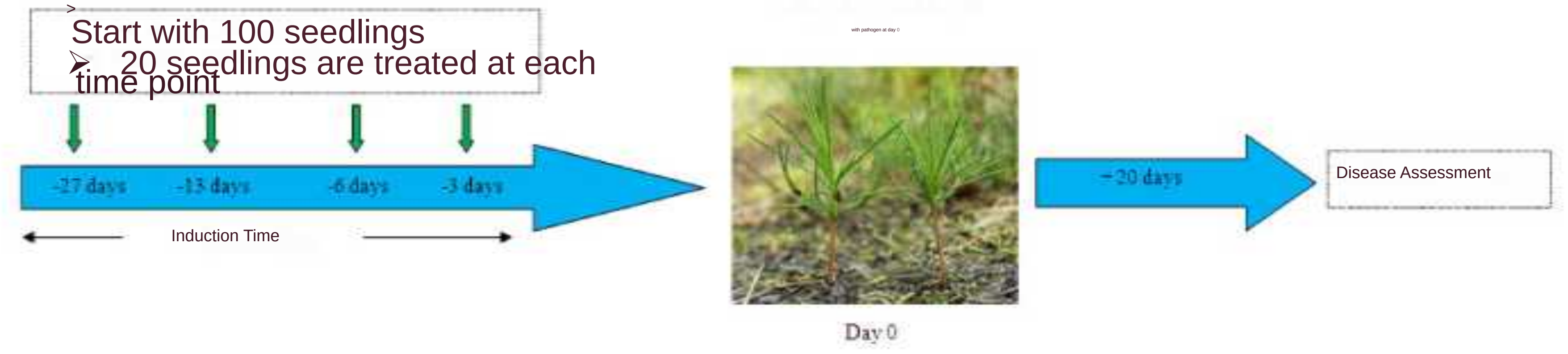


Figure: Experiment timeline on induced resistance study.

Pinus radiata (Monterey pine)

Diplodia pinea

Mathematical Model of Induced Resistance to Plant Diseases
I—Development of a Prototype Induced Resistance Model

Resistance Experiment Timeline

Figure: Greenhouse experiment: Pine seedlings were treated with Methyl jasmonate (MeJA) in with specific environmental condition (i.e. fixed temperature, moisture, light).

Mathematical Model of Induced Resistance to Plant Diseases

I—Development of a Prototype Induced Resistance Model

I—Induced Resistance Experiment Timeline

Figure: The difference between diseased (left) and healthy (right)

seedlings.

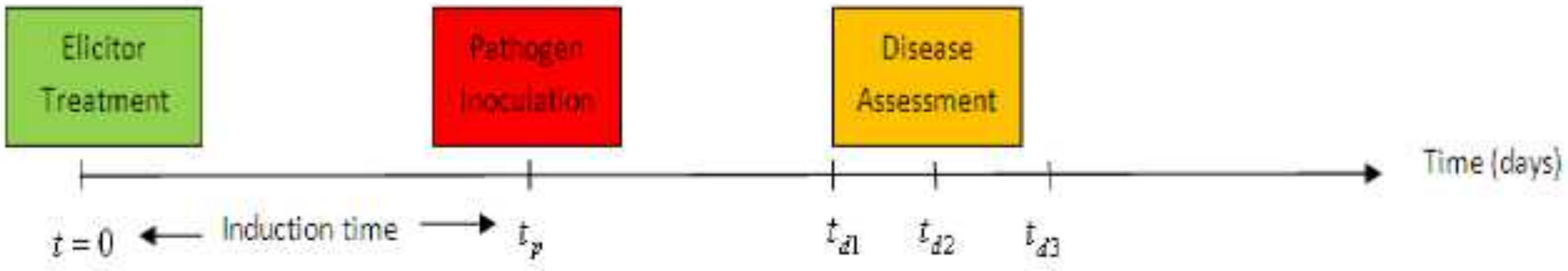


Figure: Time series on induced resistance study.

Mathematical Model of Induced Resistance to Plant Diseases ^—Development of a Prototype Induced Resistance Model

The TREATED

for $0 < t < tp$ (Pre-inoculation)

$$dR/dt = (e(t) - \gamma R)(1 - R) \qquad R(0) = B_R \qquad (1)$$

for $t_p < t < T$ (Post-inoculation)

$$dR/dt = (e(t) - \gamma R)(1 - R - D) \qquad R(t_p) = R_p \qquad (2)$$

$$dD/dt = pD(1 - R - D) \qquad D(t_p) = D_o \qquad (3)$$

where elicitor effectiveness in the system is described as;

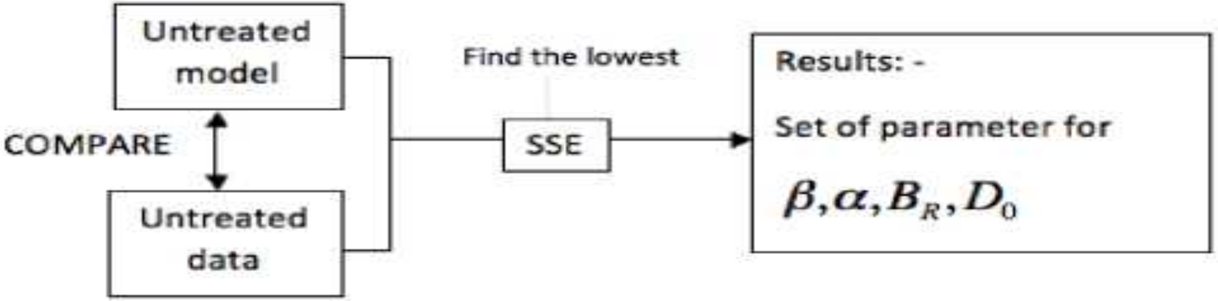
$$e(t) = \frac{kt}{t^2 + L^2}$$

and k, L are constants to be

Mathematical Model of Induced Resistance to Plant Diseases

-Parameter Estimation

|—Determine the Unknown Parameters - The Process



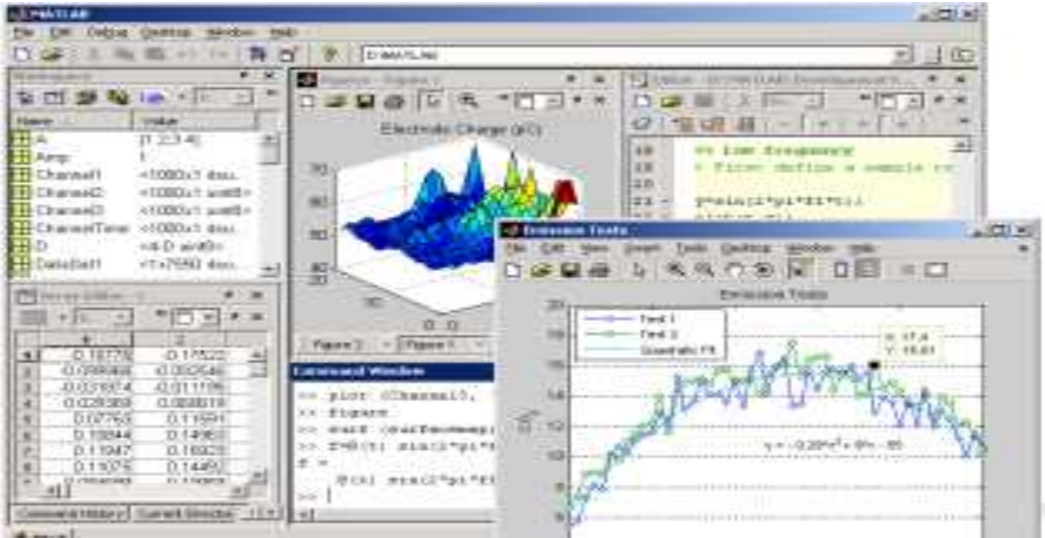


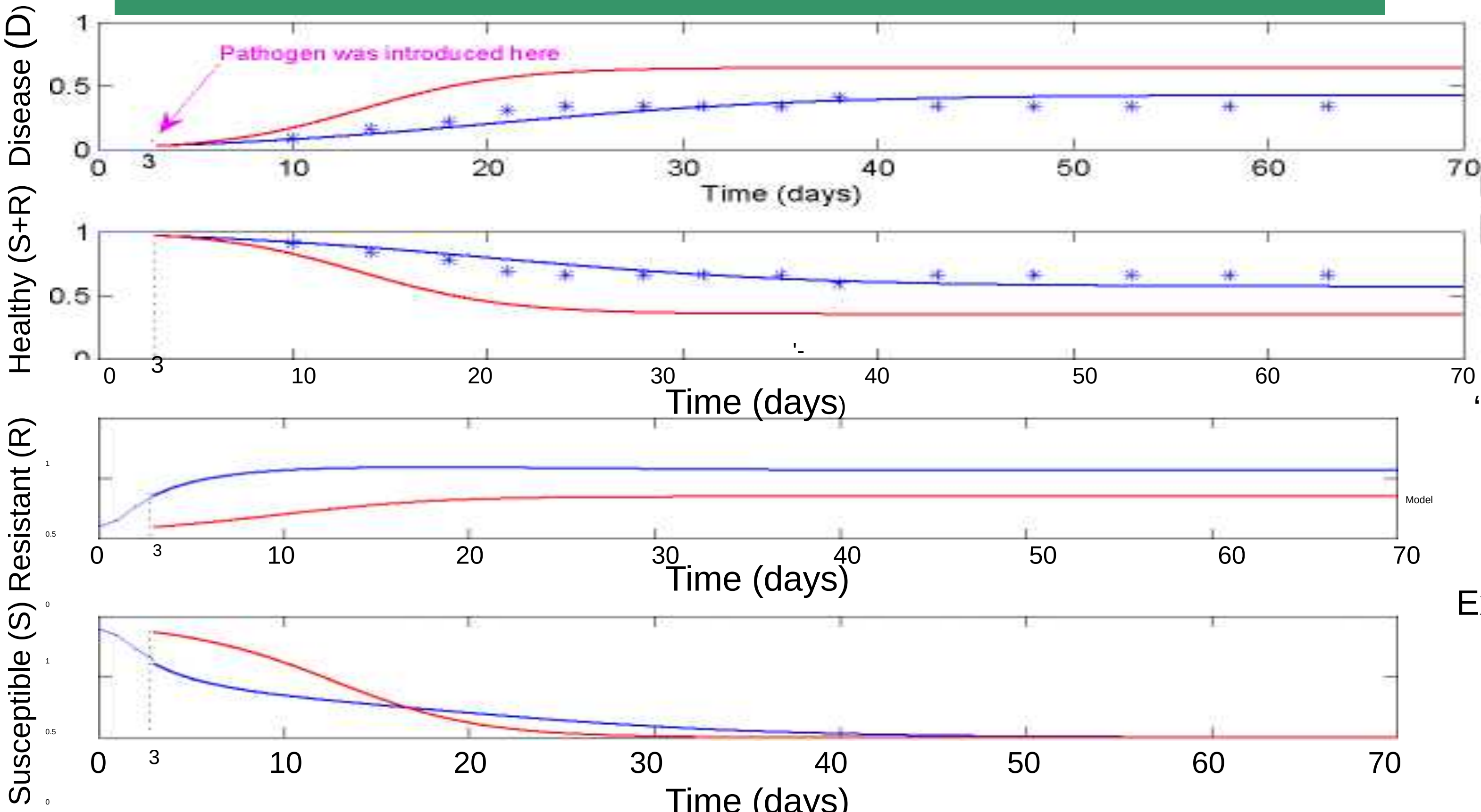
Figure: The process for parameter estimation is using MATLAB

'fminsearch'.

Mathematical Model of Induced Resistance to Plant Diseases

^—Transient Diagram with Optimal Parameters

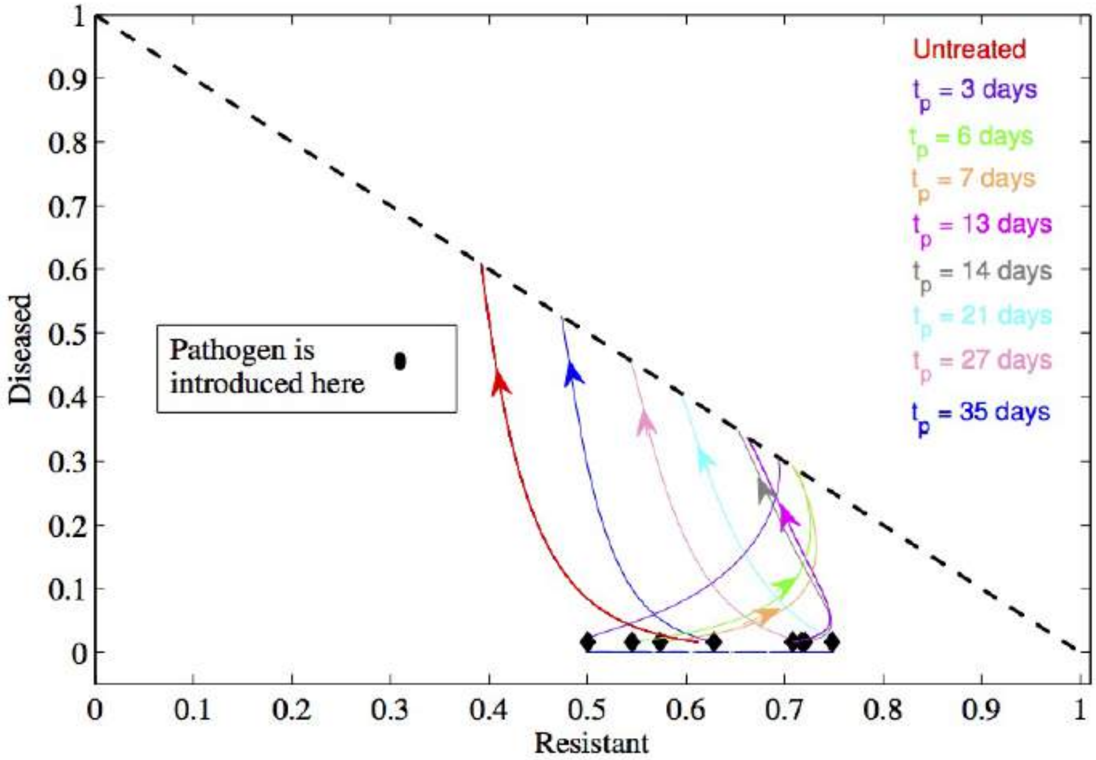
|—When inoculation time $t_p = 3$



Untreated Model in purple

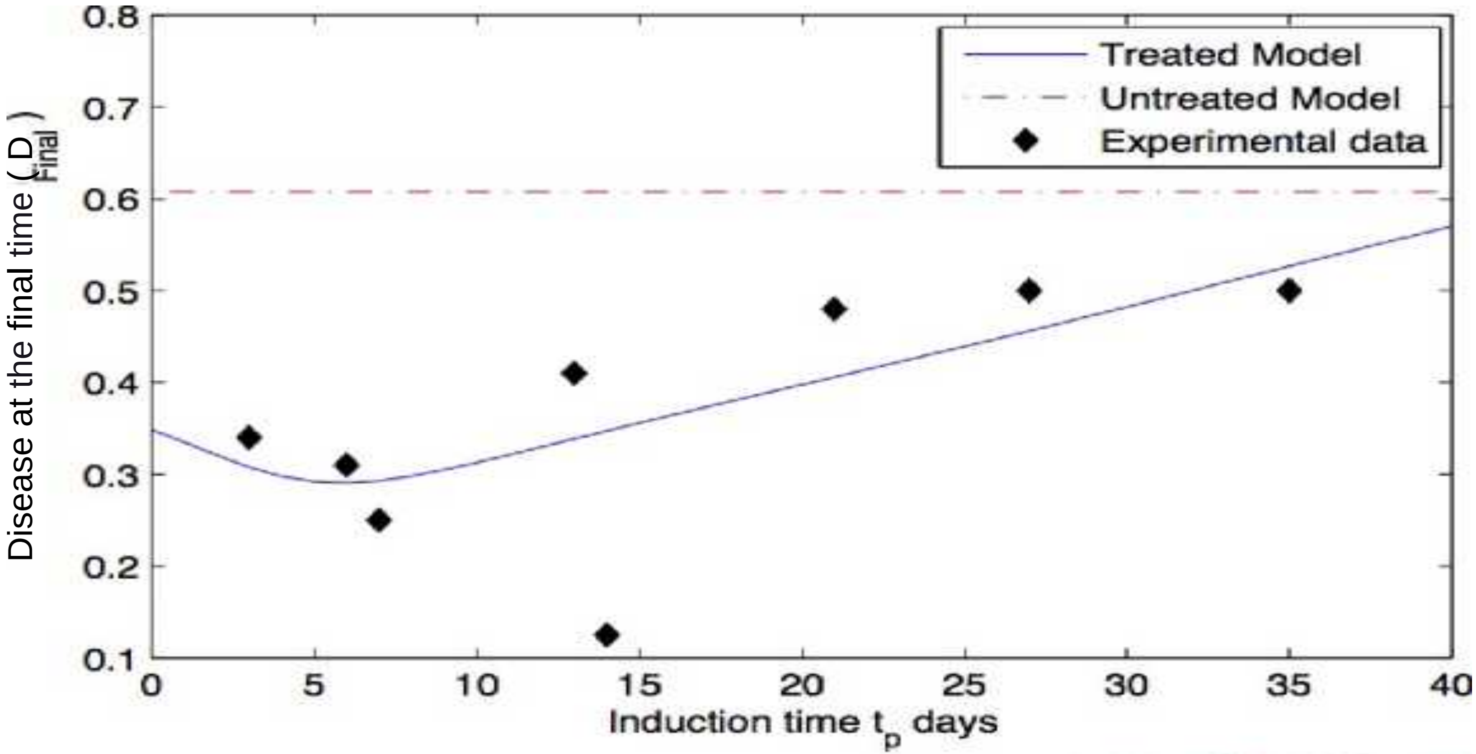
“Treated” in blue

Experimental data **


Untreated
t_p = 3 days
t_p = 6 days
t_p = 7 days
t_p = 13 days
t_p = 14 days
t_p = 21 days
t_p = 27 days
t_p = 35 days
Pathogen is introduced here
Diseased
Resistant
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

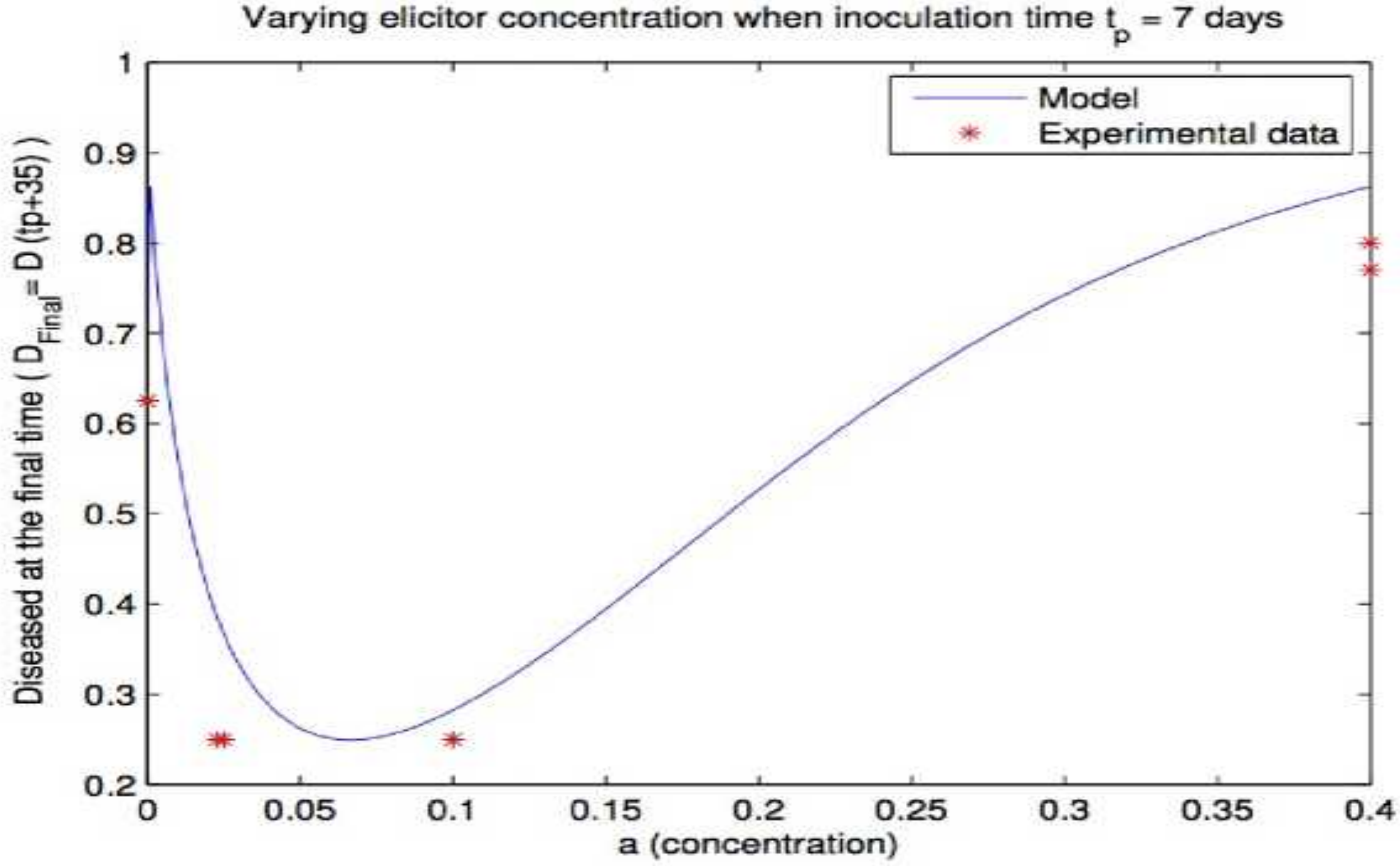
Mathematical Model of Induced Resistance to Plant Diseases

^—Determine the optimal inoculation time t_p



Mathematical Model of Induced Resistance to Plant Diseases

^—Determine the optimal elicitor concentration



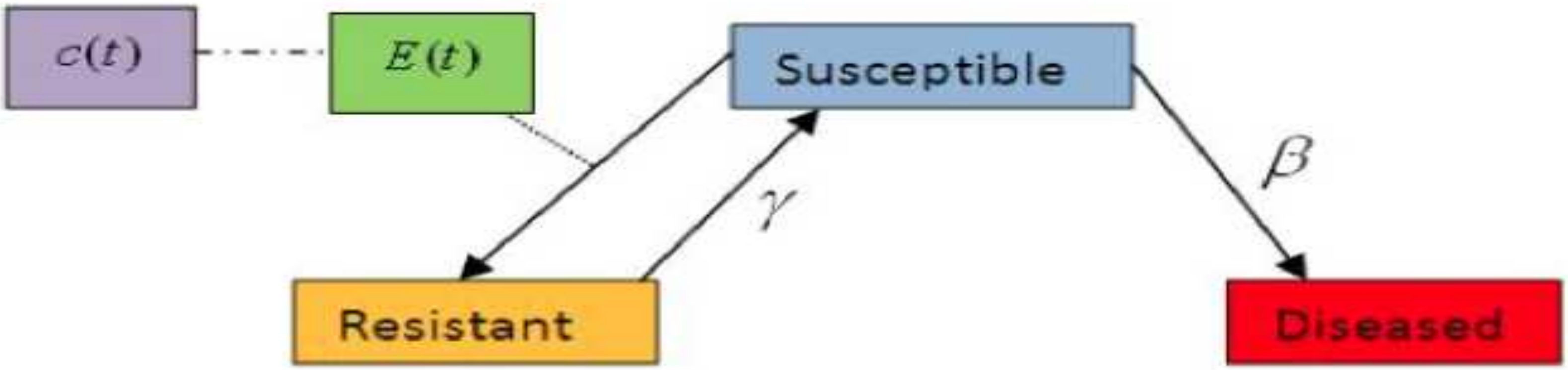


FIGURE 7.1: Schematic diagram for the control of the continuous elicitor application. The elicitor effect is determined by the elicitor applied daily until it reaches the target disease control.

The dynamics of the first stage in the interval $[0, t_p]$, with $t_p > 0$ denotes the induction time, are modelled by

$$\begin{aligned} \frac{dR}{dt} &= (E(t) - \gamma R)(1 - R) \qquad ; R(0) = R_i, \\ \frac{dE}{dt} &= -\theta E + s_0 c \qquad ; E(0) = 0. \end{aligned} \tag{7.5}$$

Whereas in the second stage $[t_p, t_f]$, the dynamics involves that of disease D:

$$\begin{aligned} \frac{dR}{dt} &= (E(t) - \gamma R)(1 - R - D) \qquad ; R(t_p) = R_p, \\ \frac{dD}{dt} &= \beta D(1 - R - D) \qquad ; D(t_p) = D_i \\ \frac{dE}{dt} &= -\theta E + s_0 c \qquad ; E(t_p) = E_p. \end{aligned} \tag{7.6}$$

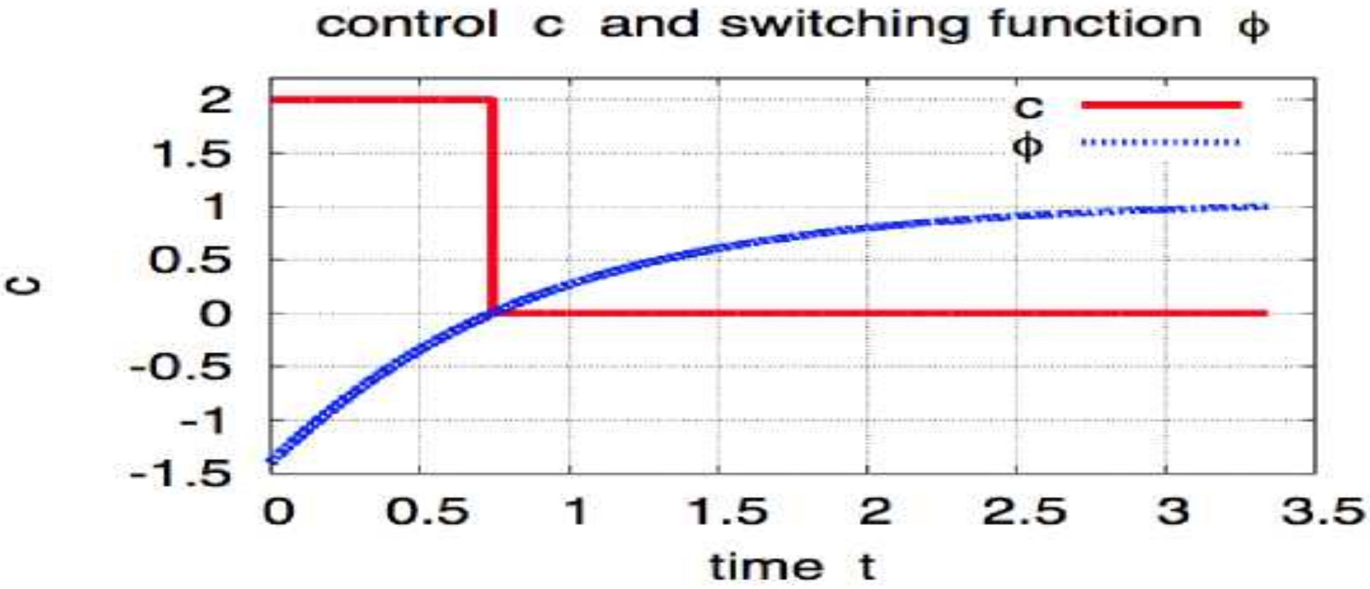
$$E(t) = \int_0^t s_0 e^{-\theta(t-\tau)} c(\tau)\, d\tau,$$

Let the control function $c(t)$ be bounded by

$$0 \leq c(t) \leq c_{max}. \tag{7.8}$$

Hence, this optimal control problem consists of determining a piecewise continuous control function $c(t)$ where $c : [0, t_f] \rightarrow [0, c_{max}]$ that minimises the objective functional

$$J\{c\} = \int_0^{t_f} c \, dt \tag{7.9}$$


control c and switching function ϕ
c
ϕ
2
1.5
1
0.5
0
-0.5
-1
-1.5
c
0
0.5
1
1.5
2
2.5
3
3.5
time t

Conclusions

- This model is able to predict the relative proportion of plants in each compartment.
- The model can be quantitatively estimate the effectiveness of elicitor treatment.
- This proposed mathematical model is generic; will be applicable for a range of plant-pathogen-elciitor scenarios.
- For more information please refer to:

^ Abdul Latif, N.S., Wake, G.C., Reglinski, T. & Elmer, P.A.G. (2014). Modelling induced resistance to plant disease. *Journal of Theoretical Biology,* 347, 144-150.

^ Nurul Syaza Abdul Latif (2014). Mathematical modelling of induced to plant disease. PhD. Dissertation. Massey University, New Zealand.

^ Abdul Latif, N.S., Wake, G.C., Reglinski, T., Elmer, P.A.G. & Taylor, J. T. (2013). Modelling induced resistance to plant disease using a dynamical system approach. *Frontiers Plant Science*, 4

What's Next??

Interested to model the epidemiology of ganoderma disease and

looking at its impact on economic losses of oil palm industry

Modelling Soil Nutrients and Pasture Yield

A commercially available model developed by Agknowledge and Wakescience

The Nutrients Model

- P is the Olsen P [μg P cm^{-3}] level in the soil
- K is the Quick Test Potassium (QTK) [cmol(+) kg^{-3}] level
- S is the extractable organic sulphur (EOS) [μg S g^{-3}]

The Nutrients Model

P= Phosphorus, K = potassium, S = Sulphur

$dP/dt =$

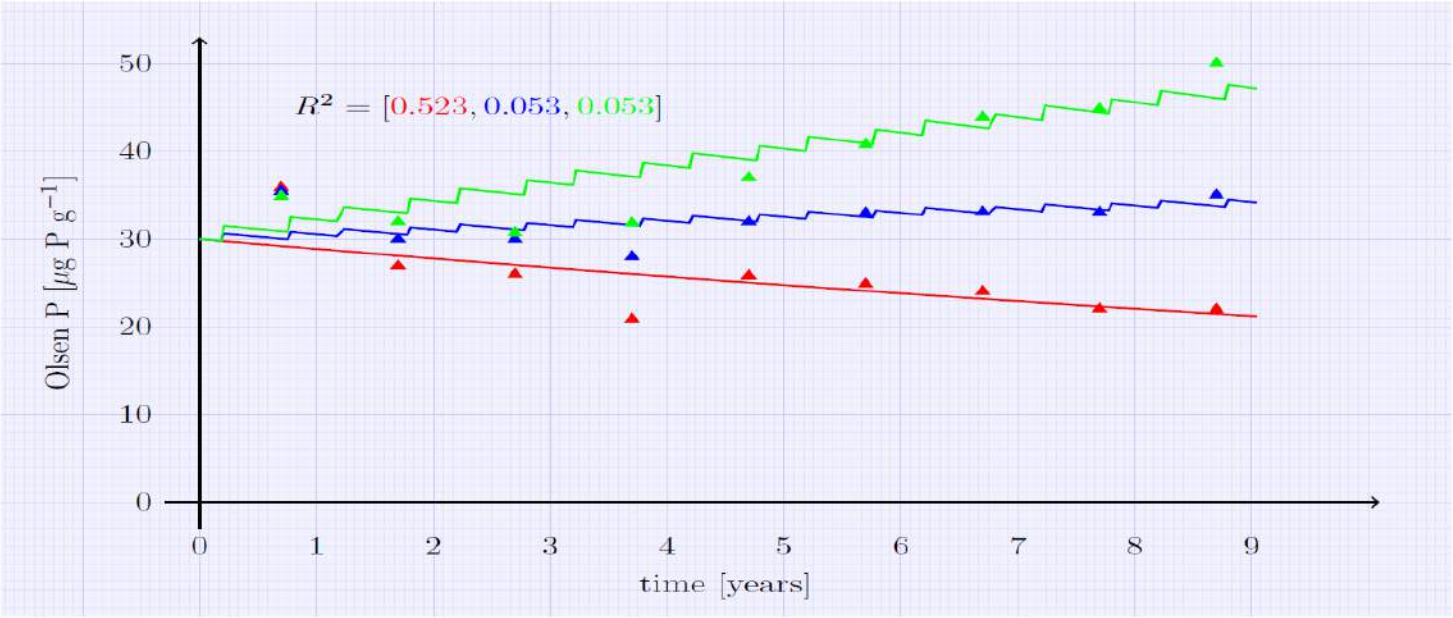
$dK/dt =$

$dS/dt =$

where the right hand sides are empirically determined functions of the constituent chemicals

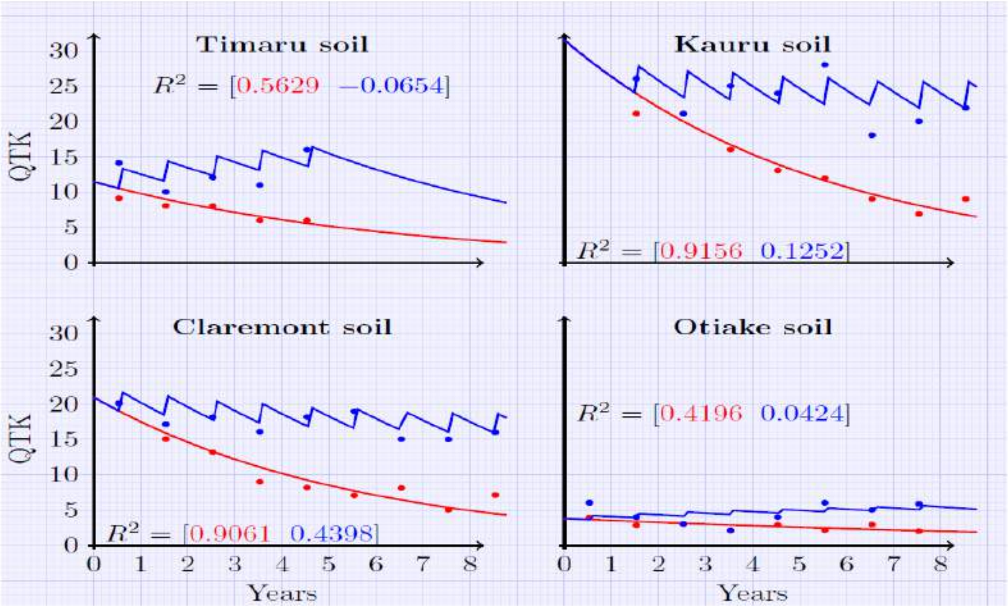
Relative yield of Grass : $dG/dt = r(P,K,S,)\ G\ (1 - G/100)$

Phosphorous



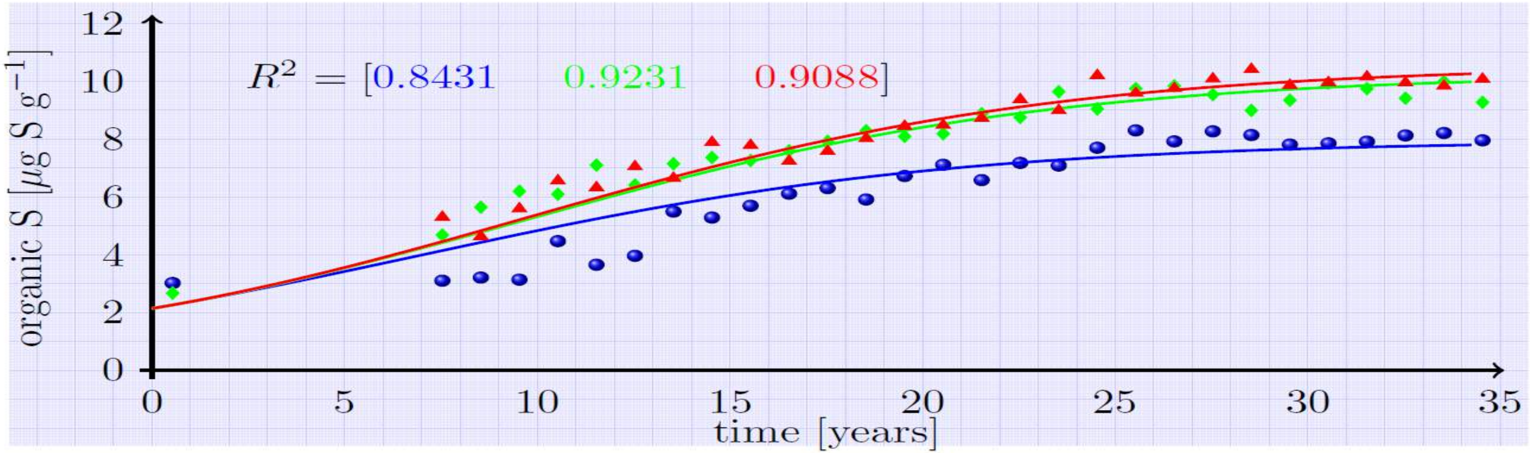
Olsen P in unfertilised ▲, and two rates of annually applied superphosphate (500 ▲ and 1000 ▲ [kg ha^{-1}]) treatments.

Potassium



Effect of application rate of 0 (●)and 60 (●) [kg K ha^{-1}] fertiliser on QTK level over time in different NZ soils.

Extractable Organic Sulphur (EOS)

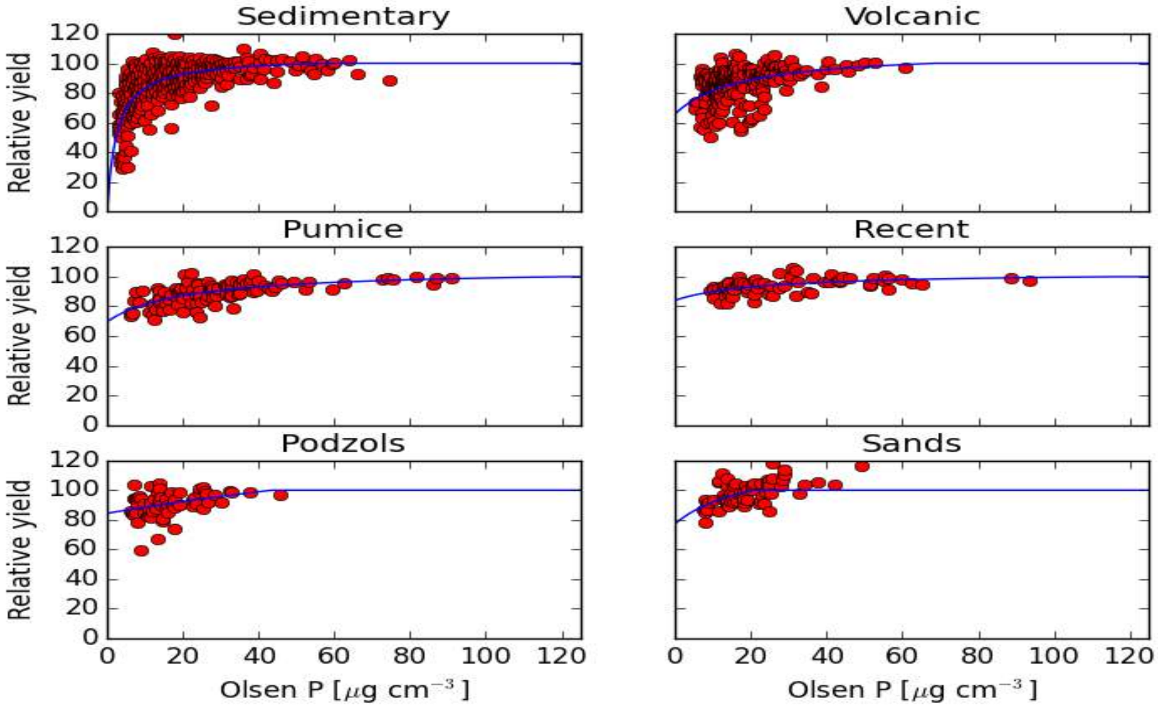


Extractable Organic Sulphur values in the soil vs time in years when the soil received superphosphate at rates 0 (●), 188 (◆), and 376 (▲) [kg ha^{-1} yr^{-1}]. The model equation is parametrized with this data and the parameters found are: $S_0 = 2.1423$, $r_s = 3.9367 \times 10^{-4}$, $K_0 = 7.9518$, and the constant c is 1.4481 for 188 [kg ha^{-1} yr^{-1}] applications and is 1.6540 for the 376 applications.

Relative Yield vs Olsen P

$$RY_{p} = a_{p}\ k^{p}{}_{1} + P / k^{p}{}_{2} + P$$

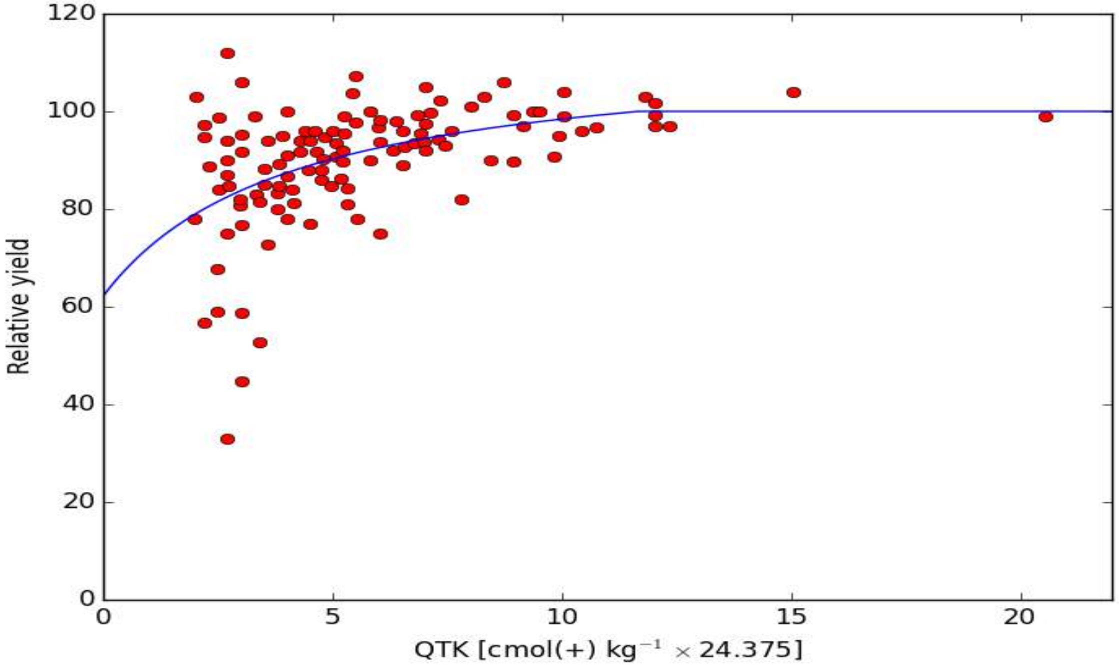
Note that different soils respond differently on same Olsen P level



Relative Yield vs QTK

$RY\downarrow k = a\downarrow k \;\; k\uparrow k \downarrow 1 + K /$
$k\uparrow k \downarrow 2 + K$

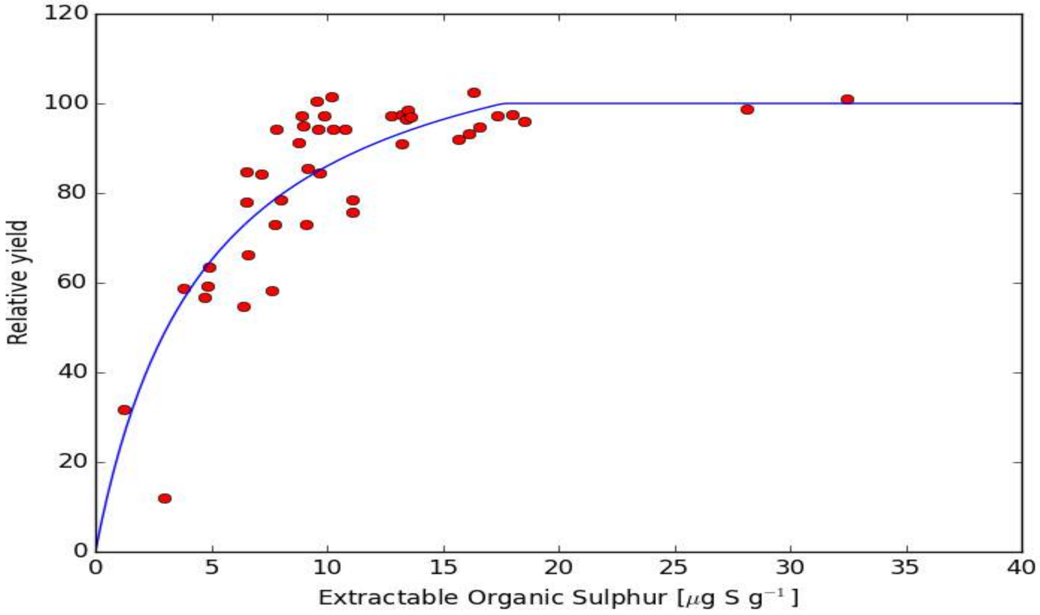
Soils have only one group for Potassium



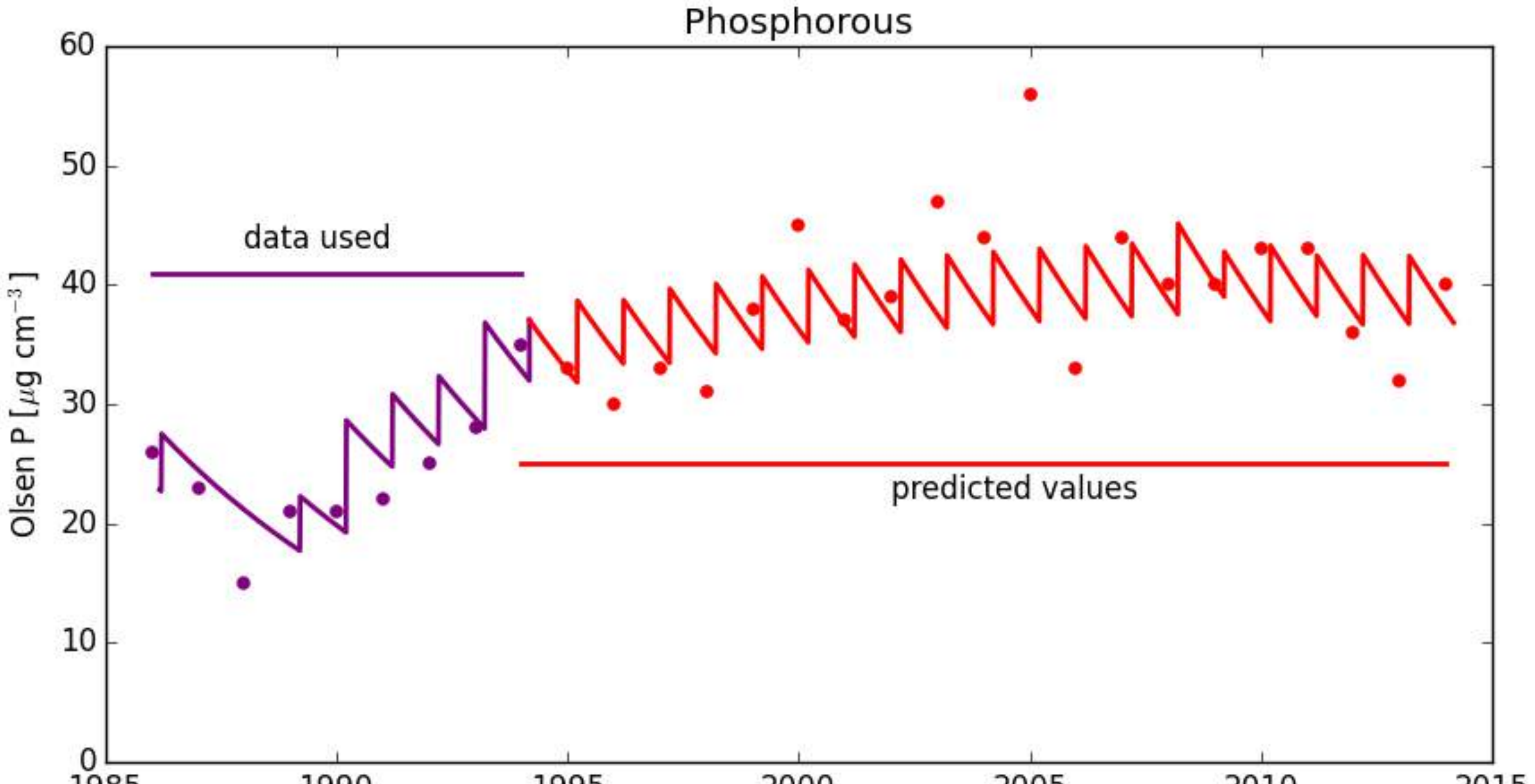
Relative Yield vs EOS

$RY_s = a_s \ k^{s}_{1} + S/ k^{s}_{2} + S$

Soils have only one group for Sulphur

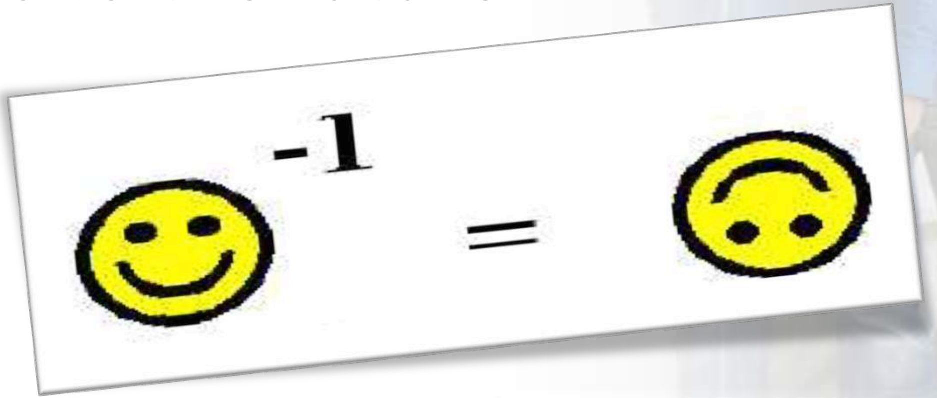


Testing of the Model



Closing Remarks

Pointers to the future




NO
WAKE
ZONE